Relevance of Risk Factors

Abstract

This paper follows the Rational Function theory that is based on the logic that asset returns being ratios of two consecutive prices are rational functions that do not add linearly in a portfolio. With reference to this theory we study the relevance of various risk factors like size, book to market ratio, investment and operating profit using the Fama-French portfolios for the US, Asia Pacific, Japan, Europe and the Global markets. The results show that the risk factors cannot definitively identify assets with higher average returns though they may help identifying assets with lower risk.

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1. Introduction.

The main objective of an asset pricing model is to estimate the asset return as accurately as possible. However, the existing factor models, like the Capital Asset Pricing Model (CAPM: Sharpe, 1964; Lintner, 1965) and the Fama French 3 and 5 Factor models (FF5F: Fama and French, 2015), also serve to indicate the risk factors that can identify assets that systematically have higher average returns compared to the other assets. Thus, the existing asset pricing models have two-fold functions: first, to estimate asset returns accurately; and second, to indicate the risk factors that capture the return variances. The empirical evidence, however, demonstrates that these models have their limitations. The average returns estimated by the CAPM tend to be lower than the actual values for lower risk assets and higher than actual values for higher risk assets (Jensen, 1968; Blume 1970; Fama and French, 1993, 1996, 2004). This led to the development of alternative asset pricing models like the Arbitrage Pricing Theory (Ross, 1976), the Fama and French (1993) three factor model, and other factor-based models (Aharoni et al, 2013; Novy-Marx, 2013; Fama and French, 2015). Out of these later models, the Fama and French five-factor model (FF5F) is now most popular among the academics. However, again, the factors of FF5F are included based on empirical evidence while their theoretical rationales remain unclear.

Thus, a gap in the literature remains in explaining the movement in asset returns, which we address by arguing that since asset returns are ratios of two consecutive asset prices, the latter themselves being functions of the market factors, the returns are rational functions. Hence asset returns do not add linearly in a portfolio as suggested by Markowitz (1952), on whose theory all the factor models are based. This means that even though the Markowitzian twin objectives of maximization of return and minimization of risk are logical, the asset returns do not add linearly for averaging purposes. For this, we have to distinguish between the average and the continuous returns, where the former has to be computed from the ratio of consecutive average prices while the latter can be computed directly from consecutive prices (Chakraborty et al., 2019).

The Rational Function Model (RFM) developed from the above argument is applicable to any set of 'fluctuating' number series that can be represented by an average series.[1] In this paper, we

[1] Thus, this approach is able to accurately estimate not only asset returns but also changes in asset volumes (Chakraborty et al., 2019). This allows us to extend the Markowitzian objective of maximizing asset returns to that of maximizing asset value (and thereby wealth) by aiming to maximize the change in the market value of the asset

try to confirm the evidence of any consistent and systematic change in the average returns due to the risk factors identified by the FF5F and also evaluate the accuracy in estimating the average asset returns. Our results indicate that the RFM estimates are consistently more accurate than the FF5F estimates to the extent that even when the FF5F estimates are negatively correlated with the actual values, the RFM estimates are positively so and well above 90%. We also find that when we compute average returns from the ratios of average prices rather than directly from the returns series, then these average returns do not exhibit any consistent pattern of change due to the risk factors identified by the FF5F, except for the Japanese market where the average returns decrease for increasing firm-size. For all the other markets consisting of the US, the Asia Pacific, Europe and the Developed category, none of the risk factors causes any consistent change in average returns. This means that we have to rethink about our perception of the risk-return relationship because the RF theory indicates that the average return and risk are not directly related. Hence, we have used the Markowitzian definition of risk to be a simple variance of returns and have modeled it on the mathematical basics pertaining to the RF theory. On testing these variance models for the US market, we find a high correlation between the estimated and the actual average variances indicating that the risk factors could help in identifying assets with lower risk.

The remainder of the paper is organized as follows. Section 2 reviews the popular asset pricing models for estimating returns and risk while Section 3 discusses the theory of the RF models. Section 4 conducts an empirical study to investigate the behavior of the average asset returns that have been sorted on the basis of four risk factors – size, book to market ratio, investment and profitability and also to compare the performance of the RFM against that of the FF5F. We also study the performance of the risk models in Section 4 while Section 5 discusses the motivation and contribution of this paper. Section 6 briefly discusses the other theoretical implications emanating from the RF theory while Section 7 concludes the paper.

2. Literature Review.

The existing literature on asset pricing has largely developed from the Sharpe-Lintner CAPM equation introduced by Sharpe (1964) and Lintner (1965). It stipulates a linear relationship between the expected return of asset i, denoted as $E(R_i)$, and the market risk $\beta_{i,m}$ and is given by:

($\Delta MV_{i,t}$) which is the arithmetic product of asset returns and change in asset volumes. This line of investigation indicates a potential source of further work.

$$E(R_i) = R_f + \beta_{i,m} [E(R_m) - R_f] \quad (1)$$

Asset i represents either an individual stock or a portfolio, while R_f and $E(R_m)$ denote the risk-free rate of return and the expected market return, respectively.

Though the CAPM appears both simple and intuitively appealing, the empirical evidence gathered subsequently highlights discrepancies, the main being that the CAPM average returns are lower than the actual average returns for lower risk assets and higher than the actual values for higher risk assets (Jensen, 1968; Blume 1970; Fama and French, 1993, 1996, 2004). This gave rise to the Joint Hypothesis Problem put forward by Fama (1970), which attributes such discrepancies to a flawed asset pricing model and/or market inefficiency. In addition, various cross-sectional studies reported systematic increase in average returns due to sorting on the basis of factors like size (Banz 1981; Fama and French, 1993), value/growth (Basu, 1977; Fama and French, 1993), momentum and reversals (Jegadeesh and Titman 1993; O' Keeffe and Gallagher, 2017), liquidity (Haugen and Baker, 1996; Datar, Naik, and Radcliffe, 1998), profitability and investment (Fama and French, 2015). These effects are not captured by the CAPM and hence they are collectively known as the CAPM anomalies. As a result, various multi-factor models came forward in order to explain these anomalies, although they often lack a sound theoretical justification (Fama and French, 1993, 2015; Novy-Marx, 2013; Hou et al. 2015).

Of the various multi-factor asset pricing models, the Fama French 5 Factor (FF5F) model is the most recent one that seemingly outperforms most other current models (Fama and French, 2015). Further, a considerable number of empirical studies have sought to investigate the explanations for these anomalies though without any conclusive success (recent examples include Erdos et al., 2011; Dempsey, 2013; Elgammal and McMillan, 2014; Elgammal et al., 2016; Bao et al., 2017). This indicates a clear research gap which this paper attempts to address through the Rational Function Model (RFM) that argues that asset returns cannot be averaged directly because returns are ratios of two consecutive asset prices, which are themselves functions of market factors. Hence, asset returns are rational functions that should be averaged not directly from returns series but through ratios of consecutive average prices (Chakraborty et al., 2019). The mathematical reasoning behind this is that for three consecutive asset prices x_{t-1}, x_t and x_{t+1}, the direct average of the time series of the returns does not equal the actual average of the time series, i.e.

$$[(x_t / x_{t-1}) + (x_{t+1}/ x_t)] \neq [(x_t + x_{t+1})/ (x_{t-1} + x_t)].$$

Similarly, for the cross-sectional returns, the direct average of two stock returns on a day 't', do not equal their index return, i.e.

$[(x_t/x_{t-1}) + (y_t/y_{t-1})] \neq [m_t/m_{t-1}]$, where $m_t = (x_t + y_t)$ and $m_{t-1} = (x_{t-1} + y_{t-1})$.

But these are the flaws of the linear asset pricing models (like the CAPM and the FF5F) when they equate the direct average return of a portfolio represented by the expected return $E(R_i)$, with the direct average return of the market portfolio, represented by the expected return $E(R_m)$, as can be seen from equation (1). It must however be clarified that the factor models hold true for the continuous returns, which are single-period returns (Chakraborty et al., 2019). This is because the continuous returns are computed as time series of daily or monthly returns based on single time intervals and so they do not exhibit the non-linear behavior of rational functions as exhibited by average returns across increasing risk. Hence, we must distinguish between multi-period average returns and single-period continuous returns and the average return should be computed from the ratio of two consecutive average prices. For this, the asset price can be estimated from the market return and the previous asset price, as is explained in the next section. Subsequently, this paper compares the RFM with the FF5F using the Fama French portfolios to study the accuracy of the two models and also to study the patterns in average returns that arise due to sorting of stocks on the basis of the Fama French factors.

In addition, we also develop two separate models for estimating risk which is taken to be the simple variance of returns (Markowitz, 1952), since the RF theory indicates that the average asset return and risk are not directly related. Of the two variance models developed from the RF theory, one model estimates the average asset variance using only the market variance whereas the other model includes additional variances based on the Fama-French risk factors. The existing literature on risk is mostly the same as that for asset returns, because, as discussed earlier, the existing asset pricing models attempt to measure both risk and return through the same factor model (Sharpe, 1964; Fama and French, 1993, 2015). Hence, we found no other comparable model for measuring asset risk separately as simple variance in returns.

3. The Rational Function Model (RFM).

Let us consider a hypothetical market where there are only three stocks trading – stocks 1, 2 and 3. Then, if their prices on day 't' are denoted by the variables $p_{1,t}$, $p_{2,t}$ and $p_{3,t}$, respectively, and if $p_{1,t}$ is correlated with $p_{2,t}$ and uncorrelated with $p_{3,t}$, we may express these relationships in linear functions (that describe them most closely) as follows:

$p_{2,t} = k' + b' (p_{1,t})$…(i);

$p_{3,t} = k'' + 0 (p_{1,t})$ …(ii); and finally

$p_{1,t} = 0 + (p_{1,t})$ …(iii).

If we assume the market combination '*m*' to be an equal weighted average of the three stock prices then averaging the equations (i), (ii) and (iii), we get:

$(p_{1,t}+p_{2,t}+p_{3,t})/3 = p_{m,t} = (k' + k'')/3 + \{(1+b')/3\}(p_{1,t})$ …(iv),

here $p_{m,t}$ is the price of the market combination '*m*'. The Equation (iv) indicates a linear relationship between $p_{m,t}$ and $p_{i,t}$ which may be generalized as

$p_{i,t} = a_i + b_i\, p_{m,t}$…(v).

This logic also holds for day 't-1' when

$p_{i,t-1} = a_i + b_i\, p_{m,t-1}$…(vi).

From equations (v) and (vi), we may deduce that the ratios $(p_{i,t}/p_{i,t-1})$ and $(p_{m,t}/p_{m,t-1})$ are also correlated which can be expressed as

$(p_{i,t}/p_{i,t-1}) = c_i + d_i\, (p_{m,t}/p_{m,t-1})$…(vii).

This gives rise to the Rational Function Model as follows:

$$p_{i,t} = \alpha_i\ (p_{i,t-1}) + \beta_i[\{(p_{m,t}/\ p_{m,t-1})p_{i,t-1}\}] +\ e_{it} \qquad (2)$$

Thus, the average return $\overline{R}_{i,t}$ is given by

$$\overline{R}_{i,t} \quad = \quad [(\overline{p}_{i,t} - \overline{p}_{i,t-1})/\ \overline{p}_{i,t-1}] \qquad (3)$$

here, $\overline{R}_{i,t}$ is the average return of an asset 'i' from days 1 to 't', whereas $\overline{p}_{i,t}$ is the average price of the same asset from days 2 to 't' while $\overline{p}_{i,t-1}$ is the average price from days 1 to 't-1'. Thus,

$$\overline{R}_{i,t} \quad = \quad [(p_{i,t} - p_{i,1}) / (p_{i,1}+\ldots+ p_{i,t-1})] \qquad (4)$$

Equation (4) indicates that the average return is dependent on the price differential and the sum of the previous prices for the time-period under study. Thus, the average return and risk are not directly related. This is a very important conclusion especially since the empirical evidence indicates that the values of β_i are all nearly 1 (Chakraborty et al., 2019), a fact which is further corroborated by the results of this study as well. It means that the assets cannot be graded for riskiness on the basis of its market coefficient β_i. However, the Markowitzian twin strategies of return maximization and risk minimization requires a measure of risk and for that we can again consider the past performance of the asset by using the variance in returns as a direct (though independent) measure of risk. The financial risk factors like size, book to market ratio, investment, profitability etc. are important for monitoring the business of a firm and would indirectly influence the returns of its stock but according to the RF theory, need not be considered as direct inputs for estimating average returns.

Moreover, as already mentioned, the RF theory being based on mathematical logic can be used to estimate the change in any set of fluctuating number series that is represented by an average index series and hence can estimate change in asset volumes as well (Chakraborty et al., 2019). However, we have not considered volume data in this paper since we have focused only on Fama French portfolios to study the FF5F risk factors. Instead, we have further extended the RF theory to estimate risk as well, which is the simple variance in asset returns. As we can see from equation (4) that the average return and risk are not directly related, so we may model risk separately. Risk being the variance in returns (Markowitz, 1952), we have extended equation (2) to approximate the variance in returns of an asset by the variance in returns of the market combination '*m*'. Thus, we have:

$$var_{i,t} = \lambda_i + \theta_i \, var_{m,t} + e_{i,t} \tag{5}$$

Here, $var_{i,t}$ is the variance in returns of asset '*i*', $var_{m,t}$ is the variance in returns of the market combination '*m*', λ_i is the constant and θ_i is the slope coefficient. For this, the time series of the variances are computed as average deviations of the returns of the asset from the RF mean return, on a rolling window basis. We have further combined the above equation (5) with other risk factors to increase the accuracy of the model for the empirical testing.

4. Empirical Study.

We consider 28 Fama French samples[2] for the US market each consisting of monthly returns from July 1963 to July 2019 (a total of 673 months) to compare the performance of the RFM with the FF5F and also to investigate the behavior of the average returns and the variances due to the FF5F sortings based on size, book-to market ratio, investment and operating profit. We feel that this 56 years' time-period is reasonably long to draw reliable and meaningful conclusions from this study. In addition, we also consider further 24 Fama French samples for Asia Pacific, Japan, Europe and the Developed markets consisting of monthly returns from July 2014 to July 2019 (i.e.

[2] We thank Kenneth French for making the data available at:
http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

the most recent 61 months in the study-period) to study the current relevance of the risk factors in international markets for identifying change in average returns.

4.1 Empirical Models

The FF5F regression equation is given by

$$R_{i,t} = R_{f,t} + \beta_{i,m} (R_{m,t}–R_{f,t}) + \beta_{i,s} SMB_t + \beta_{i,h} HML_t + \beta_{i,r} RMW_t + \beta_{i,c} CMA_t + e_{it} \quad (6)$$

Here, the expected return of asset 'i' on day 't' is denoted by $R_{i,t}$, while $R_{f,t}$ and $R_{m,t}$ denote the risk-free rate of return and the expected market return on day 't' respectively. We have ignored the intercept term used by Fama and French (1993, 2004, 2015), because it represents the unexplained variation from the average return and hence we have considered it as a part of the error term instead. Further, the factor SMB_t (for Small minus Big) is the difference between the returns of the diversified portfolios of small and big size stocks. HML_t (High minus Low) is the difference between the returns of the diversified portfolios of high and low BE/ME (ratio of book equity to market equity) stocks. RMW_t (Robust minus Weak) is the difference between the returns of the diversified portfolios of robust and weak profitability. CMA_t (Conservative minus Aggressive) is the difference between the returns of the diversified portfolios of low and high investment firms while e_{it} is a zero-mean residual term. Apart from the 52 samples giving us the values for $R_{i,t}$, the data for $R_{f,t}$, $(R_{m,t}–R_{f,t})$, SMB_t, HML_t, RMW_t and CMA_t for the various markets have also been obtained from Kenneth French's data library.

For the RFM regressions, the empirically tested parsimonious version of equation (2) for estimating asset prices (Chakraborty et al., 2019) has been used which is given by:

$$p_{i,t} = \beta_i[\{(p_{m,t}/ p_{m,t-1})\, p_{i,t-1}\}] + e_{it} \quad (7)$$

Here, the intercept is taken to be zero because there seems to be no economically justified reason for having a risk-free price of an asset and also because no asset would trade below zero. The zero-intercept model is also consistent with the empirical values of the average risk-free rate of return $R_{f,t}$ that have been negligible or zero in the literature (e.g., Mehra & Prescott, 1985). After estimating the asset prices, the average returns for the RFM are computed as per equation (3) where $\overline{p}_{i,t}$ is the average price of an asset from months 2 to 't ' and $\overline{p}_{i,t-1}$ is the average price from months 1 to 't-1'. In keeping with the RF theory, the actual average returns also have been computed as ratios of two consecutive asset prices. We have studied only average returns in this paper as they have more practical value than the continuous returns in analyzing an asset's past performance for making investment decisions.

We further estimate the risk of an asset as variance in returns from equation (5) as follows:

$$Var_1: Var_{i,t} = \theta_i \, Var_{m,t} + e_{i,t} \tag{8}$$

Here also, the intercept is taken to be zero because we logically cannot have negative values for variances. The time-series of the variances considered here are based on RF theory and are average squared deviations of the returns from the true average return computed as per RF theory on a rolling window basis. On further adding the variances of the Fama-French risk factors to equation (8), we get,

$$Var_2: Var_{i,t} = \theta_{i,m} \, Var_{m,t} + \theta_{i,s} \, Var_{SMB,t} + \theta_{i,h} \, Var_{HML,t} + \theta_{i,r} \, Var_{RMW,t} + \theta_{i,c} \, Var_{CMA,t} + e_{i,t} \tag{9}$$

Here, $Var_{SMB,t}$, $Var_{HML,t}$, $Var_{RMW,t}$, and $Var_{CMA,t}$, are the variances in returns of the corresponding Fama French factors.

4.2 Methodology

As already mentioned, the RFM has been tested against the FF5F, using 28 US samples (denoted S1 to S28) and 24 international samples (denoted S29 to S52). The details of these samples have been provided in Tables 1a and 1b. Of the 28 US samples, 8 samples are univariate quintiles, consisting of 5 portfolios that have been sorted according to one of the four FF factors, i.e. size, book-to market ratio, investment and operating profit as already mentioned earlier. Of these 8 univariate quintile samples, 4 consist of value-weighted portfolios, while the other 4 consist of equal-weighted portfolios. Similarly, there are 8 univariate decile samples consisting of 10 portfolios each, of which 4 samples are value weighted while the other 4 are equal-weighted. The remaining 12 samples are bivariate quintiles consisting of 25 portfolios each that have been obtained by bivariate quintile sorts (i.e. 5x5 sorts) for 6 double combinations from the four factors as – size and book-to-market; size and investment; size and operating profit; book-to-market and investment; book-to-market and operating profit; and operating profit and investment. For the bivariate quintile samples also, half are value weighted while the other half are equal-weighted samples. We have only considered quintiles and deciles for this study and not tertiles because dividing an entire market into three or less groups becomes too broad a categorization to draw any practically viable conclusion about whether a factor affects the asset return. Similarly, for the 24 international samples, 6 samples each are for Asia Pacific, Japan, Europe and Developed markets. Out of the 6 samples for each international market, 3 samples are value weighted while the other 3 samples are equal weighted, where these samples have been formed by bivariate quintile (5x5) sorting on size and book-to-market; size and investment; and size and operating profit.

For the RFM returns analyses, the time series of returns have been converted to time series of prices from a base price of 100. The empirical accuracy of the estimations from the FF5F and the RFM have been compared with each other based on their correlations with the actual average returns as well as their sum of squared errors (SSE).[3] For the RFM variance analyses, a time series of variances in returns from the RF means have been computed on a rolling window basis for the last 12 returns including the current return.

4.3 Empirical Results and Discussion

Tables 2a to 2d report the slopes (i.e. β_i values) and their t-statistics for the RFM regression equation (7). It can be seen from these tables that the β_i values are all positive, very close to 1.00 and highly significant. This implies that the proportionate change in the asset price $p_{i,t}$ is nearly equal to the proportionate change in market price $p_{m,t}$. Further, the fact that the R^2 values of the RFM regressions of equation (7) for all the portfolios for all the samples are consistently above 99.5% indicates that the arithmetic product of market return and the preceding asset price in equation (7) is the most important and nearly sufficient factor in estimating the asset prices. On the other hand, the R^2 values for the FF5F regressions of equation (6) lie between 53.34% to 99.07% showing that this model's accuracy is sample based which carries a possibility of erroneous estimation.

It should be mentioned here that the coefficients of FF5F regressions follow patterns more or less similar to the ones mentioned in Fama and French (2015). For example, here also the market slopes are positive and close to 1.0 and the SMB slopes are positive for small stocks and gradually decrease to negative values for the biggest stocks. Similarly, the HML slopes are negative for the low BE/ME stocks and positive for the high BE/ME stocks. The RMW slopes are negative for low profitability stocks and the CMA slopes are negative for the high investment stocks. However, these patterns do not provide any direct information about the change in average returns due to the FF5F risk factors.

The behavior of the average returns is illustrated in greater detail by Tables 3a to 3j. Table 3a shows the average returns of univariate quintiles computed as ratios of average prices while Table 3b shows direct average returns of univariate quintiles computed as direct arithmetic average of

[3] Here, we report only the important results [i.e. the regression results of Equations (7); actual average returns and the standard deviations of the various portfolios; the results of correlation analysis and the SSE values for RFM and FF5F estimates; the charts for the univariate quintiles; and the variance estimation results for the quintiles]. Other results are available upon request.

the time series of returns. Similarly, Tables 3c and 3d show average returns and their t-statistics for the univariate deciles while Tables 3e and 3f show the direct average returns and their t-statistics for the same portfolios. Fama and French (2015) have discussed only bivariate and trivariate sortings as they were more concerned about the accuracy of estimating average returns, but we have focused more on the univariate sortings because the RFM takes care of the accuracy of estimations as we shall see later and so we are more concerned about the ability of the FF5F risk factors to aid portfolio selection. As already mentioned, Tables 3a and 3b show results for univariate quintiles (samples S1 to S8) while Tables 3c to 3f show results for univariate deciles (samples S9 to S16). Tables 3g and 3h show results for bivariate quintiles (samples S17 to S28). We have not studied trivariate sortings because the trivariate sortings involve 2x4x4 splitting of the stock population whereby one factor (i.e. size) is used to divide the population into just two parts which is too broad a categorization to attribute any effect to that factor. Table 3i shows results for samples S17, S18 and S19 for the recent 5-year period (61 months) from July 2014 to July 2019 which have been studied to compare against the results for 673 months. Table 3j shows the results for the international markets for the samples S29 to S52.

Findings in Table 3a suggest that size, BE/ME ratio and investment do not seem to influence the average returns for either value-weighted or equal-weighted portfolios. Only for value-weighted profitability portfolios do the average returns increase from the lowest to the highest quintiles. On the other hand, from table 3b we may observe that the direct average returns are more influenced by the FF5F risk factors. Here, the direct average returns decrease with increasing size for both value and equal weighted portfolios while they increase with increasing BE/ME ratios and decrease with increasing investment for the equal-weighted quintiles. The direct average returns increase with increasing profitability for the value-weighted quintiles.

The picture becomes clearer by referring to the more realistic results of univariate deciles in Tables 3c and 3e. The results of deciles are more realistic than quintiles because they indicate whether the average returns change for smaller (and hence more practically relevant) variations in the FF5F risk factors. From table 3c, we may see that none of the four FF5F risk factors – size, BE/ME, investment and profitability, influence the average returns of the value-weighted or the equal-weighted deciles. Table 3e also shows that none of the FF5F risk factors influence the direct average returns of the value-weighted deciles though for equal-weighted deciles, the direct average returns increase from the lowest to the highest BE/ME stocks and decrease from the lowest to the highest investment stocks. However, as already mentioned, the average returns computed as ratios of two consecutive average prices are conceptually more accurate and so we shall accept the results of table 3c as more accurate. Hence, we may infer that the FF5F risk factors do not individually influence the average returns of assets though they might exhibit some empirical patterns for the direct average returns. This indicates that the risk factors cannot help in accurately identifying assets with higher average returns.

For the bivariate quintiles, we may see from Table 3g that none of the factors influence any of the samples in a consistent manner across all the quintiles within them. The purpose of Table 3h is mainly illustrative since direct averages are not mathematically correct measures of asset returns, but even then, on studying the results of Tables 3h and 3i for the value-weighted samples S17, S18 and S19 (the main bivariate samples as discussed in Fama and French, 2015), we can see that there are no consistent patterns across the different quintiles in the samples. Only for S19, the direct average returns generally decrease for increasing size. Table 3i samples S17*, S18* and S19* show no size, BE/ME, investment or profitability patterns. Hence, we may construe that we cannot build any definite rule about how any FF5F risk factor might influence the average returns of the assets. Similar results have been reported for the international markets in Table 3j as well, where we can see that none of the bivariate quintile samples show any consistent patterns for the average returns except for Japan, where samples S35 to S40 show a general tendency of decreasing average returns for increasing size from lowest to highest size quintiles though the other risk factors do not elicit any definite patterns of change. Thus, we conclude that contrary to the extant beliefs, the risk factors cannot be used for portfolio selection according to any definite rule, although the size factor being a very fundamental firm characteristic, may influence the asset returns in some markets.

Next, Tables 4a and 4b compare the correlations and Sum of Squared Errors (SSE) between the actual and the estimated average returns**.** Here, in Table 4a we can see that all the correlations of the RFM estimates are consistently higher than the correlations of the FF5F estimates, such that even for the samples S2, S10, S20 and S21, where the FF5F estimates are negatively correlated with the actual average returns, the RFM estimates are positively so with values above 99.5%. The SSE values of the RFM estimates are also much less than those of the FF5F estimates. These facts are borne out by the Figures 1 and 2 charts which show the plots of the actual average returns, their FF5F estimates and their RFM estimates for the value-weighted univariate quintiles (samples S1 to S4) and for equal-weighted univariate quintiles (samples S5 to S8) respectively. The results reported for the international markets in Table 4b also lead to similar conclusions, where we may again see that the correlations and the SSE values for the RFM estimates are much higher and lower respectively than those for the FF5F estimates. Hence, we may conclude that RFM provides consistent higher accuracy in estimating average returns for all portfolios, irrespective of their sorting.

For the variance analyses, we have reported the slope coefficients and their t-statistics in Tables 5 and 6 where we can see that the slope coefficients for market variance have the highest t-statistic values and hence it is the most important factor in estimating average asset variance. The average asset variance is the direct average of the time series of variances because variances being squared deviations from the mean, are additive. We can further see from the Table 6 that the FF5F factor variances also have significant t-statistic values, but their coefficients are both positive and negative indicating unpredictability of their relationship with the total asset variance. However, Tables 7a and 7b indicate that the risk factors help to improve the accuracy of the estimated average asset variance since the Sum of Squared Errors (SSEs) for the Var_2 model are generally lower than those for the Var_1 model for both US as well as other international markets.

5. Motivation and Contribution

The motivation behind this study translates into its contribution to the extant body of literature on asset pricing. The development of the RF theory and the ensuing RF model (Chakraborty et al. 2019), redefines the relationship between risk and return of an asset showing that they are not directly related. The empirical study of the RFM (Chakraborty et al. 2019) also shows that the asset returns do not increase with increasing risk whether the latter is measured in terms of variance in returns or in terms of market beta. As a result, the risk factors are not needed in estimating average asset returns. In this paper, we further study if the FF5F risk factors can help in selecting portfolios with higher average returns since, risk as measured by variance in returns in RF theory, is unable to do so. However, we find that the extant risk factors like size, book to market ratio, investment and profitability are not able to provide any definitive indication of direction of change in average asset returns. We further develop and study two risk estimation models separately and find that the above risk factors increase the accuracy of the estimated average risk of the assets. These findings indicate that though the extant risk factors are irrelevant for estimation of average returns of assets or for identifying assets with higher average returns, they might still be useful for identifying assets with lower risk, since as per the RF theory, risks and returns have to be estimated separately and then compared to select the desired portfolio.

6. Other Theoretical Implications

The theoretical foundation of the RFM has been discussed under section 3 of this paper. The direct influence of this theory is visible on the risk-return relationship of an asset which has been studied in this paper. However, there still remain a lot many other questions that can be studied subsequently. For instance, it would be interesting to know if we can generalize the RFM in terms of risk-return patterns. Or, could the RF theory be used to make reasonable forecasts of asset returns. We could use the RF theory to study the returns of other assets like those of the bond, forex and commodity markets. What implications does the RFM have on the derivatives? These are some of the many questions that can be studied in future.

Here, we discuss the possibility of generalized thumb rules for plotting the risk-return relationship of an asset. From Figures 1 and 2 it can be seen that the returns are generally highest for the middle-range risk assets and not for the highest risk assets. This may be explained by the nature of logarithmic curves which first increase or decrease sharply and then gradually taper off.

For this, we may approximate a linear relationship between the time series of standard deviations in price (x_t) and prices of an asset (p_t) as follows:

$$p_t = a_1 + b_1 x_t \tag{10}$$

Similarly, $p_{t-1} = a_2 + b_2 x_{t-1}$ (11)

Then, $ln(R_t) = ln(a_1+b_1x_t) - ln(a_2+b_2x_{t-1})$ (12)

From equation (12) above we can see that the relationship between risk and return of an asset across time may be generalized by a difference of log functions of two consecutive price variables. Since the regression results of the log returns are very similar to the regression results of arithmetic returns, the equation (12) indicates that even without actually taking logarithms, the returns being ratios of two linear equations plot logarithmically across time. However, the risk-return relationship of multiple assets cannot be mathematically captured because the overall risk is calculated across the full time-span for each asset separately and hence different assets in a collection of assets, have different overall risks. This indicates that the risk-return profiling of the assets should be done on a case-to-case basis for selecting the most mean-variance efficient asset. For this, we may define the Mean-Variance Efficiency Quotient (MVEQ) to be the ratio of the average return and the overall risk for each asset as follows:

Mean Variance Efficiency Quotient (MVEQ) = (Mean Return)/(Overall Variance in Returns) (13)

The higher the value of MVEQ, the more efficient the asset would be, i.e. it would have higher return for lower risk. Hence, it would be advisable to invest in assets having high MVEQ values.

7. Summary and Conclusions.

The extant empirical literature reports discrepancies between the CAPM estimates and the actual average returns. It also documents patterns of increasing or decreasing average returns exhibited by portfolios sorted on various financial factors that are called as anomalies. The factor models seek to include these financial factors to account for the unexplained variations in average returns but with limited success in improving the accuracy of estimation and no success in explaining the theoretical reasons behind the discrepancies. Based on mathematical logic, we argue that these discrepancies are arising because of the erroneous practice of taking direct averages of returns across cross-sections and across time since returns being rational functions (ratios of two consecutive prices which themselves are functions of market factors) do not add linearly. We derive our Rational Function model (RFM) from this mathematical logic combined with the economic relationship that connects the rate of change in asset prices with the rate of change in index prices. We conduct an empirical study to compare the performance of the RFM with that of the FF5F and also to investigate the anomalous patterns reported for the FF5F risk factors to ascertain if they can be used to choose assets with higher average returns. We find that the FF5F risk factors do not exhibit any definite pattern for the average returns that are consistent across different portfolios and different time periods, especially when the average returns are computed as ratios of average prices. However, size, being a fundamental firm characteristic, might influence the average returns and the variances in some markets which should be verified specifically before using this factor. We also find that the RFM estimates of average returns are consistently quite accurate even for samples which generate FF5F estimates that are negatively correlated with the actual average returns. These findings lead us to conclude that the RF theory is both theoretically and empirically sound and hence risk and average return are not directly related and both the parameters should be separately estimated. Thus, the FF5F risk factors are not definite indicators for identifying assets that have systematically higher average returns though they may help identify assets with lower average variances in returns, i.e. less risky assets. Hence, portfolio selection should be primarily based on the past performance of the stocks in terms of prices, volumes and variances in returns and also on the market movement. Though financial risk factors may indirectly influence the market valuation of the stocks by indicating the business performance of the firm, they should not be taken as direct inputs for portfolio selection. We hope that the RFM and the findings of this study would stimulate further studies on this subject in future that would add to the empirical evidence in this new light.

Conflict of Interest: The authors declare that they have no conflict of interest.

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Table 1a: Sample Description for the US market from July 1963 to July 2019

S. No.	Type of Sample	Number of Portfolios	Weightage of Portfolios	Number of Months	Sorting Factor(s)	Sample Name
1	Fama-French Univariate Quintiles	5	Value	673	Size	S1
2	Fama-French Univariate Quintiles	5	Value	673	Book to Market	S2
3	Fama-French Univariate Quintiles	5	Value	673	Investment	S3
4	Fama-French Univariate Quintiles	5	Value	673	Operating Profit	S4
5	Fama-French Univariate Quintiles	5	Equal	673	Size	S5
6	Fama-French Univariate Quintiles	5	Equal	673	Book to Market	S6
7	Fama-French Univariate Quintiles	5	Equal	673	Investment	S7
8	Fama-French Univariate Quintiles	5	Equal	673	Operating Profit	S8
9	Fama-French Univariate Deciles	10	Value	673	Size	S9
10	Fama-French Univariate Deciles	10	Value	673	Book to Market	S10
11	Fama-French Univariate Deciles	10	Value	673	Investment	S11
12	Fama-French Univariate Deciles	10	Value	673	Operating Profit	S12
13	Fama-French Univariate Deciles	10	Equal	673	Size	S13
14	Fama-French Univariate Deciles	10	Equal	673	Book to Market	S14
15	Fama-French Univariate Deciles	10	Equal	673	Investment	S15
16	Fama-French Univariate Deciles	10	Equal	673	Operating Profit	S16
17	Fama-French Bivariate Quintiles	25	Value	673	Size and Book to Market	S17
18	Fama-French Bivariate Quintiles	25	Value	673	Size and Investment	S18
19	Fama-French Bivariate Quintiles	25	Value	673	Size and Operating Profit	S19
20	Fama-French Bivariate Quintiles	25	Value	673	Book to Market and Investment	S20
21	Fama-French Bivariate Quintiles	25	Value	673	Book to Market and Operating Profit	S21
22	Fama-French Bivariate Quintiles	25	Value	673	Operating Profit and Investment	S22
23	Fama-French Bivariate Quintiles	25	Equal	673	Size and Book to Market	S23
24	Fama-French Bivariate Quintiles	25	Equal	673	Size and Investment	S24
25	Fama-French Bivariate Quintiles	25	Equal	673	Size and Operating Profit	S25
26	Fama-French Bivariate Quintiles	25	Equal	673	Book to Market and Investment	S26
27	Fama-French Bivariate Quintiles	25	Equal	673	Book to Market and Operating Profit	S27
28	Fama-French Bivariate Quintiles	25	Equal	673	Operating Profit and Investment	S28

Table 1b: Sample Description for other markets from July 2014 to July 2019

S. No.	Type of Sample	Number of Portfolios	Weightage of Portfolios	Number of Months	Sorting Factor(s)	Sample Name
1	Fama-French Bivariate Quintiles - Asia Pacific	25	Value	61	Size and Book to Market	S29
2	Fama-French Bivariate Quintiles - Asia Pacific	25	Value	61	Size and Investment	S30
3	Fama-French Bivariate Quintiles - Asia Pacific	25	Value	61	Size and Operating Profit	S31
4	Fama-French Bivariate Quintiles - Asia Pacific	25	Equal	61	Size and Book to Market	S32
5	Fama-French Bivariate Quintiles - Asia Pacific	25	Equal	61	Size and Investment	S33
6	Fama-French Bivariate Quintiles - Asia Pacific	25	Equal	61	Size and Operating Profit	S34
7	Fama-French Bivariate Quintiles - Japan	25	Value	61	Size and Book to Market	S35
8	Fama-French Bivariate Quintiles - Japan	25	Value	61	Size and Investment	S36
9	Fama-French Bivariate Quintiles - Japan	25	Value	61	Size and Operating Profit	S37
10	Fama-French Bivariate Quintiles - Japan	25	Equal	61	Size and Book to Market	S38
11	Fama-French Bivariate Quintiles - Japan	25	Equal	61	Size and Investment	S39
12	Fama-French Bivariate Quintiles - Japan	25	Equal	61	Size and Operating Profit	S40
13	Fama-French Bivariate Quintiles - Europe	25	Value	61	Size and Book to Market	S41
14	Fama-French Bivariate Quintiles - Europe	25	Value	61	Size and Investment	S42
15	Fama-French Bivariate Quintiles - Europe	25	Value	61	Size and Operating Profit	S43
16	Fama-French Bivariate Quintiles - Europe	25	Equal	61	Size and Book to Market	S44
17	Fama-French Bivariate Quintiles - Europe	25	Equal	61	Size and Investment	S45
18	Fama-French Bivariate Quintiles - Europe	25	Equal	61	Size and Operating Profit	S46
19	Fama-French Bivariate Quintiles - Global	25	Value	61	Size and Book to Market	S47
20	Fama-French Bivariate Quintiles - Global	25	Value	61	Size and Investment	S48
21	Fama-French Bivariate Quintiles - Global	25	Value	61	Size and Operating Profit	S49
22	Fama-French Bivariate Quintiles - Global	25	Equal	61	Size and Book to Market	S50
23	Fama-French Bivariate Quintiles - Global	25	Equal	61	Size and Investment	S51
24	Fama-French Bivariate Quintiles - Global	25	Equal	61	Size and Operating Profit	S52

Table 2a: Slope coefficients and t-stats of the RFM Equation (7) for Univariate Quintiles for the US market:

$$p_{i,t} = \beta_i[\{(p_{m,t}/ p_{m,t-1})\, p_{i,t-1}\}] + e_{it}$$

S. No.	Samples	1	2	3	4	5	1	2	3	4	5
		β_i					t-stats				
1	S1	0.9963	0.9978	0.9983	0.9995	0.9983	849.39	1033.62	1416.02	2173.58	4125.97
2	S2	1.0005	0.9989	0.9980	0.9963	0.9971	2293.33	2520.23	1944.80	1358.26	1044.88
3	S3	0.9974	0.9976	0.9981	0.9988	0.9998	1796.77	2075.98	2516.72	2562.61	1327.16
4	S4	0.9957	0.9976	0.9986	0.9985	0.9997	947.54	2193.57	2550.51	2902.88	2312.77
5	S5	0.9950	0.9975	0.9979	0.9988	0.9983	860.93	987.85	1273.42	1844.65	2802.84
6	S6	0.9962	0.9977	0.9970	0.9963	0.9954	768.91	1216.71	1209.97	1142.13	953.94
7	S7	0.9950	0.9972	0.9983	0.9977	0.9941	789.45	1387.42	1523.25	1449.30	761.60
8	S8	0.9950	0.9980	0.9980	0.9977	0.9972	719.85	1327.93	1429.37	1540.33	1390.88

Tables 2b and 2c: Slope coefficients and t-stats of the RFM Equation (7) for Univariate Deciles for the US market:

$$p_{i,t} = \beta_i[\{(p_{m,t}/p_{m,t-1})\, p_{i,t-1}\}] + e_{it}$$

S. No.	Samples	1	2	3	4	5	6	7	8	9	10
		β_i									
1	S9	0.9956	0.9968	0.9982	0.9974	0.9977	0.9987	0.9989	0.9998	0.9985	0.9982
2	S10	1.0009	1.0000	0.9998	0.9977	0.9987	0.9972	0.9950	0.9977	0.9972	0.9969
3	S11	0.9969	0.9980	0.9981	0.9972	0.9980	0.9980	0.9979	0.9996	1.0015	0.9965
4	S12	0.9949	0.9960	0.9967	0.9976	1.0002	0.9971	0.9964	1.0004	1.0001	0.9992
5	S13	0.9945	0.9962	0.9978	0.9971	0.9975	0.9984	0.9983	0.9993	0.9983	0.9981
6	S14	0.9947	0.9972	0.9983	0.9970	0.9978	0.9964	0.9962	0.9963	0.9963	0.9944
7	S15	0.9938	0.9973	0.9972	0.9972	0.9979	0.9986	0.9977	0.9977	0.9976	0.9914
8	S16	0.9939	0.9971	0.9980	0.9980	0.9982	0.9979	0.9977	0.9977	0.9975	0.9969

S. No.	Samples	1	2	3	4	5	6	7	8	9	10
		t-stats									
1	S9	872.66	788.86	989.72	1036.60	1185.95	1520.21	1667.76	2278.51	2699.47	3002.15
2	S10	1661.92	1757.91	2161.70	1773.18	1797.18	1474.62	1204.72	1146.78	1168.17	771.56
3	S11	1411.91	1611.66	1523.63	1811.36	1896.19	1925.81	1931.23	1759.16	1244.67	1004.41
4	S12	598.69	1216.09	1540.32	1670.55	1868.29	1891.54	2055.74	2126.28	1603.01	1908.92
5	S13	823.78	782.80	942.94	1013.33	1102.94	1417.76	1530.95	1995.73	2271.89	2350.43
6	S14	606.14	1006.54	1167.25	1159.01	1230.42	1121.05	1135.05	1105.59	1106.31	781.19
7	S15	654.18	1023.03	1280.49	1387.85	1427.00	1529.91	1489.67	1349.02	1130.16	603.64
8	S16	554.17	1205.60	1303.13	1289.18	1332.40	1455.02	1445.37	1537.45	1394.49	1265.61

Table 2d: Slope coefficients and t-stats of the RFM Equation (7) for Bivariate Quintiles for the US market:

$$p_{i,t} = \beta_i[\{(p_{m,t}/ p_{m,t-1})\, p_{i,t-1}\}] + e_{it}$$

S. No.	Samples	βi					t-stats				
		1	2	3	4	5	1	2	3	4	5
1	S17										
	i	0.9920	0.9980	0.9991	1.0020	1.0004	426.23	678.69	800.67	1300.38	1972.31
	ii	0.9976	1.0003	0.9992	1.0001	0.9986	671.37	1016.41	1413.17	1866.05	1842.77
	iii	0.9963	0.9983	0.9983	0.9980	0.9978	899.43	1065.11	1296.44	1354.62	1538.82
	iv	0.9965	0.9964	0.9970	0.9964	0.9950	882.11	904.24	1143.56	1397.07	909.68
	v	0.9958	0.9958	0.9958	0.9969	0.9968	859.08	840.23	874.37	938.31	768.49
2	S18										
	i	0.9956	0.9964	0.9981	0.9973	0.9969	663.21	823.27	1015.58	1402.63	1377.18
	ii	0.9965	0.9983	0.9977	0.9986	0.9972	884.55	1053.54	1319.27	1838.77	1686.88
	iii	0.9981	0.9978	0.9983	0.9998	0.9978	929.20	1022.19	1377.16	1993.84	2003.72
	iv	0.9970	0.9988	0.9982	0.9998	0.9986	939.48	1045.12	1363.77	1890.35	1914.65
	v	0.9941	0.9965	0.9985	0.9998	1.0005	643.22	724.68	913.70	914.91	1107.13
3	S19										
	i	0.9952	0.9969	0.9976	0.9976	0.9946	591.14	608.04	612.20	765.19	870.33
	ii	0.9977	0.9976	0.9977	1.0001	0.9968	928.79	1071.98	1208.50	1591.42	1619.82
	iii	0.9967	0.9983	0.9988	0.9998	0.9982	904.85	1010.65	1301.64	1819.68	1881.06
	iv	0.9966	0.9963	0.9981	0.9993	0.9983	845.02	966.05	1393.22	1811.02	2228.50
	v	0.9951	0.9967	0.9978	0.9996	0.9998	746.03	946.93	1226.43	1872.74	1869.02
4	S20										
	i	0.9997	0.9967	0.9960	0.9969	0.9954	1426.79	1027.12	1165.78	876.48	862.19
	ii	0.9978	0.9992	0.9981	0.9948	0.9967	1324.17	1363.62	1228.18	985.23	914.71
	iii	0.9984	0.9979	0.9982	0.9975	0.9976	1452.68	1566.98	1289.36	1215.33	778.44
	iv	0.9997	0.9979	0.9996	0.9969	0.9975	1292.98	1549.49	1438.52	845.09	817.21
	v	1.0034	1.0022	0.9927	0.9946	0.9954	873.67	1184.65	832.24	879.98	736.10
5	S21										
	i	0.9963	0.9960	0.9958	0.9964	0.9955	449.17	724.89	901.26	979.17	939.51
	ii	1.0063	0.9983	0.9970	0.9960	0.9978	623.94	1074.28	1248.14	1181.88	865.17
	iii	1.0005	0.9994	0.9992	0.9973	0.9969	737.84	1343.82	1480.81	990.79	871.74
	iv	1.0014	0.9985	0.9977	0.9949	0.9939	1440.92	1656.93	1187.42	872.12	538.24
	v	0.9998	0.9976	0.9962	0.9938	0.9918	1906.05	1190.79	765.83	513.78	396.57
6	S22										
	i	0.9959	0.9945	0.9971	0.9978	1.0005	769.67	981.76	1088.43	1023.76	1399.16
	ii	0.9944	0.9969	0.9971	0.9990	0.9977	920.54	1211.28	1142.00	1306.38	1300.34
	iii	0.9964	0.9975	0.9997	0.9975	0.9978	858.46	1175.26	1544.94	1537.12	1255.95
	iv	0.9998	0.9980	0.9978	0.9984	0.9984	823.65	1373.36	1454.49	1564.73	1087.79
	v	0.9925	0.9987	0.9993	1.0001	1.0021	630.28	809.25	993.10	1022.40	836.29

Table 2d (contd.): Slope coefficients and t-stats of the RFM Equation (7) for Bivariate Quintiles for the US market:

$$p_{i,t} = \beta_i[\{(p_{m,t}/ p_{m,t-1})\, p_{i,t-1}\}] + e_{it}$$

S. No.	Samples	βi					t-stats				
		1	2	3	4	5	1	2	3	4	5
7	S23										
	i	0.9916	0.9978	0.9986	1.0012	0.9994	452.26	671.97	822.98	1279.43	1937.98
	ii	0.9948	0.9996	0.9989	0.9996	0.9990	706.44	1019.65	1295.35	1690.09	2115.33
	iii	0.9951	0.9985	0.9985	0.9977	0.9984	851.74	1008.29	1207.93	1291.77	1720.48
	iv	0.9959	0.9963	0.9968	0.9962	0.9962	977.43	873.74	1091.56	1321.63	1121.74
	v	0.9952	0.9950	0.9955	0.9960	0.9959	879.97	808.25	786.62	853.15	909.51
8	S24										
	i	0.9940	0.9960	0.9972	0.9966	0.9978	666.15	793.29	886.99	1254.30	1642.50
	ii	0.9955	0.9987	0.9979	0.9982	0.9982	947.15	1013.60	1251.25	1672.09	1997.84
	iii	0.9972	0.9979	0.9984	0.9994	0.9985	1047.38	1034.31	1354.60	1892.60	2085.25
	iv	0.9960	0.9989	0.9977	0.9997	0.9991	1038.48	1025.07	1219.37	1742.01	2615.88
	v	0.9925	0.9950	0.9972	0.9981	0.9970	639.72	691.04	832.27	888.21	1023.21
9	S25										
	i	0.9941	0.9963	0.9970	0.9960	0.9948	681.85	644.77	649.44	744.26	722.72
	ii	0.9975	0.9977	0.9978	0.9997	0.9978	1058.66	1033.30	1175.77	1697.26	1496.21
	iii	0.9968	0.9982	0.9986	0.9988	0.9991	1080.91	979.03	1193.74	1613.50	2228.41
	iv	0.9963	0.9961	0.9982	0.9991	0.9982	899.71	926.76	1281.24	1646.57	2320.31
	v	0.9942	0.9964	0.9973	0.9993	0.9989	735.77	894.22	1102.53	1695.21	2230.66
10	S26										
	i	0.9945	0.9953	0.9954	0.9949	0.9947	556.81	749.64	757.78	715.66	715.96
	ii	0.9990	0.9964	0.9975	0.9971	0.9957	1462.61	1250.11	1265.47	1148.18	953.83
	iii	1.0008	0.9996	0.9974	0.9976	0.9965	1478.61	1581.72	1252.61	1232.49	998.93
	iv	0.9997	0.9980	0.9980	0.9961	0.9958	1331.95	1391.20	1232.15	1117.10	990.56
	v	0.9915	0.9973	0.9957	0.9949	0.9936	571.92	806.86	873.53	891.87	760.67
11	S27										
	i	0.9904	0.9948	0.9950	0.9955	0.9946	374.91	505.83	644.90	845.88	849.31
	ii	0.9997	0.9995	0.9993	0.9971	0.9970	624.52	1321.80	1349.65	1127.68	1037.99
	iii	1.0028	0.9997	0.9972	0.9972	0.9973	858.12	1662.98	1304.92	1078.46	951.67
	iv	0.9986	0.9986	0.9975	0.9949	0.9940	1447.64	1639.05	1249.36	908.20	622.97
	v	0.9989	0.9966	0.9946	0.9937	0.9896	1720.03	1063.68	714.64	571.84	391.42
12	S28										
	i	0.9943	0.9966	0.9962	0.9948	0.9964	643.25	998.23	1087.21	968.84	1018.92
	ii	0.9953	0.9974	0.9983	0.9981	0.9968	929.51	1235.91	1286.41	1321.62	1296.01
	iii	0.9974	0.9981	0.9982	0.9989	0.9986	996.74	1275.65	1357.28	1521.73	1547.63
	iv	0.9955	0.9983	0.9982	0.9983	0.9971	872.83	1256.81	1369.44	1413.63	1291.50
	v	0.9901	0.9970	0.9976	0.9964	0.9971	480.33	1029.29	1058.52	1137.21	1013.52

Table 3a: Average Standard Deviations of Returns, Actual Average Returns and their t statistics for Univariate Quintiles for the US market:

S. No.	Samples	$\bar{\sigma}_i$ ()	1	2	3	4	5	1	2	3	4	5
			$\bar{R}_{i,t}$ (%)					t-stats				
1	S1	5.31	0.61	0.71	0.75	0.79	0.60	2.54	3.17	3.65	4.09	3.70
2	S2	4.62	0.74	0.71	0.65	0.46	0.61	4.06	4.14	3.97	2.70	3.03
3	S3	4.56	0.70	0.60	0.65	0.64	0.68	3.80	3.91	4.11	3.80	3.20
4	S4	4.63	0.37	0.54	0.64	0.66	0.76	1.74	3.15	3.85	3.94	4.45
5	S5	5.58	0.58	0.71	0.73	0.74	0.65	2.36	3.04	3.38	3.69	3.68
6	S6	5.80	0.47	0.72	0.72	0.69	0.70	1.79	3.23	3.50	3.47	3.11
7	S7	5.73	0.66	0.80	0.84	0.74	0.35	2.54	4.19	4.48	3.62	1.35
8	S8	5.63	0.53	0.82	0.79	0.78	0.72	1.97	4.06	4.10	3.95	3.23

Table 3b: Actual Direct Average Returns and their t statistics for Univariate Quintiles for the US market:

S. No.	Samples	1	2	3	4	5	1	2	3	4	5
		$\bar{R}'_{i,t}$ (%) (Direct Average of Returns Series)					t-stats				
1	S1	0.97	0.96	0.91	0.85	0.63	4.07	4.29	4.44	4.42	3.87
2	S2	0.73	0.71	0.69	0.70	0.92	4.00	4.15	4.21	4.06	4.60
3	S3	0.89	0.70	0.67	0.72	0.74	4.83	4.54	4.24	4.26	3.45
4	S4	0.57	0.58	0.68	0.70	0.82	2.66	3.35	4.09	4.14	4.84
5	S5	1.14	0.95	0.91	0.84	0.69	4.62	4.05	4.19	4.18	3.89
6	S6	0.70	0.98	1.04	1.16	1.44	2.69	4.39	5.04	5.84	6.37
7	S7	1.42	1.14	1.08	1.05	0.68	5.41	5.97	5.72	5.19	2.62
8	S8	1.07	1.08	1.01	1.06	1.09	3.96	5.37	5.26	5.33	4.85

Tables 3c and 3d: Average Standard Deviations of Returns, Actual Average Returns and their t statistics for Univariate Deciles for the US market:

S. No.	Samples	$\bar{\sigma_i}$ ()	1	2	3	4	5	6	7	8	9	10
			$\bar{R}_{i,t}$ (%)									
1	S9	5.38	0.57	0.63	0.77	0.66	0.71	0.78	0.76	0.80	0.70	0.58
2	S10	4.76	0.76	0.73	0.81	0.59	0.68	0.63	0.38	0.56	0.64	0.53
3	S11	4.71	0.65	0.74	0.63	0.59	0.68	0.62	0.60	0.69	0.83	0.46
4	S12	4.77	0.34	0.41	0.48	0.57	0.74	0.55	0.51	0.80	0.77	0.74
5	S13	5.61	0.57	0.59	0.76	0.66	0.70	0.78	0.73	0.76	0.69	0.61
6	S14	5.81	0.36	0.61	0.76	0.67	0.77	0.68	0.71	0.67	0.73	0.65
7	S15	5.70	0.56	0.86	0.82	0.78	0.83	0.85	0.73	0.75	0.66	0.18
8	S16	5.58	0.44	0.75	0.82	0.81	0.81	0.76	0.78	0.79	0.74	0.70

S. No.	Samples	1	2	3	4	5	6	7	8	9	10
		t-stats									
1	S9	2.38	2.60	3.37	2.99	3.31	3.90	3.86	4.20	3.98	3.58
2	S10	3.91	4.13	4.67	3.34	3.97	3.78	2.17	3.16	3.37	2.25
3	S11	3.19	4.09	3.85	3.80	4.25	3.75	3.61	3.82	4.05	1.99
4	S12	1.38	2.10	2.60	3.27	4.22	3.23	2.98	4.62	4.51	4.25
5	S13	2.26	2.36	3.17	2.86	3.10	3.71	3.53	3.81	3.72	3.49
6	S14	1.30	2.52	3.36	3.04	3.65	3.33	3.54	3.33	3.48	2.64
7	S15	1.91	3.87	4.16	4.14	4.41	4.45	3.67	3.59	2.88	0.64
8	S16	1.47	3.48	3.95	4.11	4.21	3.94	3.94	3.91	3.44	3.01

Tables 3e and 3f: Actual Direct Average Returns and their t statistics for Univariate Deciles for the US market:

S. No.	Samples	1	2	3	4	5	6	7	8	9	10
		$\bar{R}'_{i,t}$ (%) (Direct Average of Returns Series)									
1	S9	1.00	0.94	1.00	0.93	0.95	0.89	0.88	0.83	0.73	0.61
2	S10	0.72	0.76	0.76	0.68	0.66	0.74	0.64	0.76	0.90	0.97
3	S11	0.91	0.88	0.73	0.69	0.68	0.68	0.74	0.71	0.84	0.61
4	S12	0.61	0.57	0.57	0.60	0.73	0.64	0.62	0.76	0.88	0.78
5	S13	1.20	0.90	0.98	0.91	0.93	0.89	0.87	0.81	0.74	0.64
6	S14	0.63	0.86	0.96	1.00	1.03	1.05	1.16	1.16	1.31	1.55
7	S15	1.49	1.31	1.18	1.10	1.08	1.07	1.07	1.04	0.95	0.53
8	S16	1.07	1.09	1.11	1.05	1.04	0.98	1.05	1.07	1.08	1.09

S. No.	Samples	1	2	3	4	5	6	7	8	9	10
		t-stats									
1	S9	4.16	3.90	4.36	4.20	4.43	4.41	4.46	4.35	4.18	3.76
2	S10	3.73	4.27	4.37	3.81	3.85	4.47	3.68	4.29	4.76	4.12
3	S11	4.49	4.89	4.47	4.44	4.26	4.13	4.44	3.93	4.09	2.62
4	S12	2.46	2.88	3.08	3.50	4.17	3.79	3.62	4.42	5.14	4.45
5	S13	4.79	3.63	4.10	3.96	4.13	4.21	4.22	4.10	4.00	3.69
6	S14	2.27	3.54	4.23	4.52	4.90	5.13	5.85	5.75	6.24	6.27
7	S15	5.11	5.91	6.02	5.85	5.73	5.64	5.39	4.99	4.16	1.87
8	S16	3.59	5.07	5.36	5.31	5.41	5.07	5.31	5.32	5.04	4.66

Table 3g: Average Standard Deviations of Returns, Actual Average Returns and their t statistics for Bivariate Quintiles for the US market:

S. No.	Samples ($\bar{\sigma}_i$)	$\bar{R}_{i,t}$ (%)					t-stats				
		1	2	3	4	5	1	2	3	4	5
1	S17 (5.54)										
	i	0.22	0.70	0.62	0.70	0.68	0.72	2.64	2.73	3.24	2.95
	ii	0.63	0.90	0.81	0.63	0.57	2.32	3.91	3.89	3.14	2.46
	iii	0.70	0.84	0.76	0.72	0.66	2.79	4.06	3.97	3.80	3.03
	iv	0.94	0.84	0.65	0.64	0.58	4.17	4.30	3.42	3.49	2.69
	v	0.74	0.67	0.62	0.32	0.50	4.17	3.98	3.79	1.80	2.44
2	S18 (5.43)										
	i	0.62	0.74	0.85	0.66	0.31	2.26	3.44	3.92	2.88	1.16
	ii	0.65	0.76	0.81	0.85	0.52	2.68	3.80	4.01	3.91	1.95
	iii	0.78	0.76	0.82	0.77	0.67	3.55	4.10	4.47	3.78	2.68
	iv	0.68	0.75	0.87	0.84	0.73	3.32	4.11	4.84	4.46	3.03
	v	0.64	0.54	0.60	0.60	0.73	3.64	3.56	3.81	3.55	3.49
3	S19 (5.47)										
	i	0.45	0.79	0.68	0.75	0.54	1.63	3.57	3.18	3.38	2.13
	ii	0.55	0.69	0.81	0.70	0.76	2.07	3.16	3.99	3.29	3.21
	iii	0.59	0.73	0.80	0.77	0.83	2.34	3.68	4.21	3.91	3.71
	iv	0.55	0.83	0.79	0.83	0.85	2.36	4.22	4.34	4.44	4.18
	v	0.27	0.44	0.58	0.62	0.74	1.28	2.59	3.50	3.71	4.47
4	S20 (5.11)										
	i	0.81	0.61	0.66	0.69	0.93	3.86	3.40	3.79	3.81	3.99
	ii	0.67	0.81	0.66	0.65	0.91	3.48	4.65	3.81	3.45	4.30
	iii	0.55	0.70	0.70	0.81	0.30	2.89	4.14	4.12	4.48	1.37
	iv	0.55	0.42	0.60	0.50	0.32	2.74	2.46	3.47	2.41	1.55
	v	0.52	0.58	0.65	0.71	0.28	2.32	2.84	2.98	3.09	1.15
5	S21 (5.48)										
	i	0.28	1.04	0.70	0.80	0.74	0.91	4.14	3.17	4.24	4.23
	ii	0.40	0.69	0.70	0.75	0.64	1.68	3.56	3.77	4.36	3.40
	iii	0.40	0.55	0.76	0.62	0.70	1.96	3.14	4.49	3.46	3.34
	iv	0.38	0.46	0.64	0.32	0.28	1.91	2.61	3.69	1.59	1.11
	v	0.43	0.61	0.65	0.57	0.53	1.99	3.04	3.01	2.25	1.64
6	S22 (5.08)										
	i	0.45	0.39	0.49	0.57	0.16	1.85	1.87	2.34	2.56	0.62
	ii	0.39	0.58	0.58	0.70	0.56	2.01	3.30	3.21	3.70	2.53
	iii	0.70	0.65	0.74	0.58	0.64	3.59	3.89	4.38	3.12	2.84
	iv	0.86	0.70	0.59	0.68	0.76	4.48	4.40	3.49	3.78	3.45
	v	0.96	0.63	0.66	0.61	0.94	5.01	3.65	3.92	3.41	4.14

Table 3g (contd.): Average Standard Deviations of Returns, Actual Average Returns and their t statistics for Bivariate Quintiles for the US market:

S. No.	Samples ($\bar{\sigma}_i$)	$\bar{R}_{i,t}$ (%)					t-stats				
		1	2	3	4	5	1	2	3	4	5
7	S23 (5.77)										
	i	0.21	0.51	0.62	0.69	0.71	0.66	1.90	2.55	3.11	3.01
	ii	0.61	0.88	0.87	0.63	0.56	2.12	3.71	4.04	3.06	2.32
	iii	0.67	0.85	0.79	0.72	0.66	2.55	3.91	3.96	3.72	2.87
	iv	0.89	0.80	0.65	0.64	0.56	3.84	3.98	3.30	3.35	2.43
	v	0.70	0.74	0.71	0.49	0.56	3.56	4.17	4.07	2.75	2.71
8	S24 (5.64)										
	i	0.61	0.74	0.81	0.61	0.24	2.12	3.38	3.73	2.66	0.87
	ii	0.64	0.84	0.86	0.89	0.44	2.53	4.06	4.13	4.02	1.56
	iii	0.78	0.82	0.82	0.77	0.57	3.37	4.25	4.33	3.63	2.18
	iv	0.67	0.75	0.85	0.83	0.60	3.12	3.97	4.58	4.28	2.41
	v	0.71	0.72	0.72	0.72	0.49	3.86	4.41	4.32	4.06	2.18
9	S25 (5.67)										
	i	0.50	0.84	0.72	0.76	0.52	1.76	3.76	3.37	3.37	2.02
	ii	0.52	0.76	0.84	0.70	0.74	1.85	3.40	4.05	3.15	2.97
	iii	0.55	0.75	0.82	0.81	0.79	2.06	3.68	4.15	3.94	3.37
	iv	0.44	0.84	0.75	0.84	0.82	1.80	4.20	3.92	4.35	3.89
	v	0.32	0.60	0.74	0.67	0.76	1.43	3.16	4.28	3.84	4.25
10	S26 (5.86)										
	i	0.47	0.81	0.93	0.77	0.17	1.51	3.59	4.32	3.44	0.59
	ii	0.70	0.67	0.89	0.76	0.59	2.65	3.39	4.53	3.61	2.26
	iii	0.68	0.78	0.79	0.75	0.53	2.73	4.17	4.28	3.84	2.19
	iv	0.68	0.74	0.83	0.67	0.45	2.80	4.14	4.61	3.39	1.92
	v	0.75	0.80	0.75	0.66	0.34	2.91	3.82	3.53	2.97	1.33
11	S27 (6.08)										
	i	0.12	0.67	0.92	0.70	0.78	0.35	2.35	3.71	3.10	3.48
	ii	0.49	0.85	0.84	0.83	0.72	1.61	3.78	4.18	4.21	3.18
	iii	0.56	0.87	0.71	0.78	0.65	2.07	4.42	3.77	4.05	2.81
	iv	0.65	0.72	0.77	0.56	0.24	2.70	3.85	4.28	2.75	0.88
	v	0.61	0.86	0.83	0.76	0.35	2.50	4.10	3.92	3.07	1.13
12	S28 (5.64)										
	i	0.62	0.69	0.74	0.50	0.09	2.12	3.05	3.17	2.01	0.29
	ii	0.74	0.84	0.90	0.86	0.58	3.37	4.53	4.71	4.30	2.40
	iii	0.82	0.82	0.84	0.82	0.62	3.88	4.79	4.82	4.23	2.60
	iv	0.77	0.85	0.87	0.79	0.60	3.54	4.74	4.93	4.00	2.54
	v	0.84	0.71	0.84	0.68	0.62	3.56	3.48	4.24	3.26	2.48

Table 3h: Actual Direct Average Returns and their t statistics for Bivariate Quintiles for the US market:

S. No.	Samples	$\bar{R}'_{i,t}$ (%) (Direct Average of Returns Series)					t-stats				
		1	2	3	4	5	1	2	3	4	5
1	S17										
	i	0.60	1.06	1.00	1.17	1.29	2.00	4.02	4.37	5.38	5.61
	ii	0.80	1.00	1.05	1.03	1.11	2.95	4.38	5.05	5.09	4.81
	iii	0.81	1.00	0.89	0.96	1.11	3.21	4.82	4.67	5.13	5.13
	iv	0.90	0.80	0.82	0.90	0.91	4.01	4.10	4.30	4.89	4.20
	v	0.72	0.67	0.62	0.52	0.69	4.10	3.95	3.80	2.95	3.35
2	S18										
	i	1.19	1.14	1.17	1.09	0.65	4.31	5.29	5.41	4.80	2.41
	ii	1.07	1.04	1.06	1.09	0.77	4.41	5.18	5.26	5.00	2.90
	iii	1.05	1.03	0.94	0.99	0.79	4.78	5.54	5.09	4.85	3.18
	iv	0.91	0.84	0.88	0.93	0.82	4.46	4.62	4.90	4.92	3.42
	v	0.78	0.63	0.62	0.66	0.73	4.47	4.12	3.92	3.93	3.48
3	S19										
	i	0.83	1.14	1.06	1.16	1.06	3.00	5.11	4.91	5.25	4.20
	ii	0.85	0.97	0.98	0.97	1.17	3.19	4.45	4.81	4.54	4.91
	iii	0.79	0.89	0.89	0.92	1.14	3.13	4.50	4.69	4.68	5.10
	iv	0.74	0.82	0.81	0.90	1.01	3.18	4.17	4.42	4.81	4.94
	v	0.48	0.48	0.62	0.63	0.77	2.29	2.82	3.73	3.78	4.64
4	S20										
	i	0.76	0.72	0.75	0.75	0.86	3.64	4.00	4.32	4.15	3.67
	ii	0.85	0.87	0.61	0.75	0.76	4.44	5.01	3.52	4.02	3.60
	iii	0.77	0.73	0.76	0.77	0.59	4.05	4.31	4.44	4.27	2.69
	iv	0.85	0.65	0.75	0.83	0.74	4.17	3.76	4.30	3.98	3.56
	v	1.09	0.90	0.81	1.00	0.73	4.88	4.42	3.73	4.39	3.01
5	S21										
	i	0.51	0.72	0.74	0.72	0.79	1.65	2.88	3.36	3.78	4.54
	ii	0.58	0.70	0.68	0.76	0.80	2.40	3.59	3.67	4.42	4.21
	iii	0.50	0.59	0.72	0.83	1.02	2.46	3.38	4.23	4.65	4.87
	iv	0.61	0.64	0.83	0.79	0.94	3.05	3.59	4.77	3.99	3.71
	v	0.83	0.90	1.01	1.03	1.14	3.88	4.47	4.63	4.09	3.51
6	S22										
	i	0.89	0.59	0.73	0.66	0.38	3.65	2.84	3.47	2.98	1.53
	ii	0.73	0.70	0.52	0.75	0.63	3.77	3.98	2.86	3.95	2.85
	iii	0.83	0.82	0.67	0.72	0.65	4.27	4.88	4.00	3.87	2.91
	iv	1.06	0.81	0.64	0.74	0.71	5.57	5.04	3.79	4.14	3.24
	v	1.06	0.78	0.79	0.79	0.98	5.52	4.48	4.68	4.38	4.32

Table 3h (contd.): Actual Direct Average Returns and their t statistics for Bivariate Quintiles for the US market:

S. No.	Samples		$\bar{R}'_{i,t}$ (%) (Direct Average of Returns Series)					t-stats				
			1	2	3	4	5	1	2	3	4	5
7	S23											
		i	0.71	1.11	1.18	1.32	1.56	2.24	4.12	4.88	5.94	6.59
		ii	0.77	1.02	1.10	1.04	1.10	2.68	4.28	5.13	4.99	4.60
		iii	0.80	1.01	0.92	1.00	1.15	3.02	4.66	4.60	5.14	5.03
		iv	0.88	0.82	0.83	0.91	0.95	3.80	4.03	4.22	4.78	4.10
		v	0.74	0.75	0.73	0.63	0.78	3.76	4.18	4.23	3.50	3.78
8	S24											
		i	1.57	1.34	1.27	1.16	0.69	5.42	6.10	5.84	5.04	2.47
		ii	1.06	1.09	1.10	1.14	0.70	4.18	5.26	5.31	5.12	2.52
		iii	1.13	1.06	0.96	1.02	0.73	4.87	5.54	5.05	4.82	2.77
		iv	0.93	0.87	0.90	0.92	0.76	4.35	4.63	4.88	4.77	3.06
		v	0.88	0.76	0.74	0.73	0.62	4.76	4.64	4.46	4.12	2.77
9	S25											
		i	1.19	1.29	1.17	1.29	1.11	4.18	5.81	5.48	5.69	4.33
		ii	0.82	1.03	1.01	0.99	1.11	2.92	4.63	4.86	4.50	4.46
		iii	0.79	0.90	0.92	0.95	1.12	2.98	4.40	4.67	4.64	4.78
		iv	0.70	0.86	0.83	0.92	0.98	2.87	4.29	4.35	4.77	4.65
		v	0.57	0.62	0.71	0.72	0.82	2.55	3.31	4.08	4.14	4.60
10	S26											
		i	1.08	0.90	0.94	0.92	0.47	3.46	3.99	4.36	4.10	1.65
		ii	1.28	1.01	1.02	1.04	0.82	4.86	5.12	5.17	4.97	3.15
		iii	1.27	1.02	1.01	0.98	0.95	5.07	5.49	5.43	5.00	3.89
		iv	1.44	1.06	1.12	1.14	1.03	5.94	5.92	6.21	5.76	4.34
		v	1.71	1.42	1.30	1.31	1.04	6.66	6.81	6.13	5.92	4.10
11	S27											
		i	0.57	0.83	0.80	0.79	0.95	1.68	2.88	3.25	3.50	4.23
		ii	1.04	0.92	0.91	0.98	1.18	3.41	4.07	4.50	4.97	5.24
		iii	1.03	1.00	0.94	1.09	1.30	3.78	5.05	5.04	5.65	5.62
		iv	1.21	1.02	1.12	1.26	1.35	4.99	5.51	6.19	6.23	5.03
		v	1.44	1.42	1.38	1.71	1.32	5.96	6.78	6.55	6.91	4.24
12	S28											
		i	1.48	1.19	1.08	0.91	0.46	5.06	5.27	4.62	3.65	1.54
		ii	1.31	1.14	1.17	1.07	0.81	6.00	6.14	6.14	5.36	3.35
		iii	1.32	1.03	0.99	1.07	0.84	6.25	6.00	5.66	5.54	3.53
		iv	1.44	1.12	1.04	1.08	0.88	6.63	6.27	5.89	5.50	3.68
		v	1.54	1.21	1.14	1.13	0.91	6.52	5.95	5.76	5.43	3.61

Table 3i: Actual Direct Average Returns for three Samples of Bivariate Quintiles from the US market over smaller time-period (July 2014 to July 2019):

S. No.	Samples	$\bar{R}'_{i,t}$ (%) (Direct Average of Returns Series)				
		1	2	3	4	5
1	S17*					
	i	0.378	0.617	0.452	0.480	0.288
	ii	0.864	0.987	0.636	0.446	0.154
	iii	0.958	0.694	0.642	0.415	0.193
	iv	1.088	0.834	0.746	0.264	0.402
	v	1.093	0.750	0.653	0.284	0.483
2	S18*					
	i	0.155	0.488	0.721	0.541	0.269
	ii	0.340	0.676	0.663	0.772	0.658
	iii	0.521	0.483	0.632	0.629	0.787
	iv	0.371	0.740	0.884	0.840	0.855
	v	0.512	0.643	0.586	0.706	1.241
3	S19*					
	i	0.266	0.721	0.533	0.467	0.180
	ii	0.811	0.635	0.677	0.300	0.489
	iii	0.729	0.638	0.706	0.557	0.466
	iv	0.511	1.001	0.856	0.744	0.798
	v	0.205	0.641	0.852	0.723	0.921

Table 3j: Average Standard Deviations of Returns, Actual Average Returns and their t statistics for the international markets:

S. No.	Samples	($\bar{\sigma_i}$)	$\bar{R}_{i,t}$ (%)					t-stats				
			1	2	3	4	5	1	2	3	4	5
1	S29	(4.72)										
		i	0.47	0.04	0.02	0.18	0.37	0.50	0.07	0.03	0.30	0.54
		ii	-0.49	-0.46	-0.52	-0.19	0.17	-0.77	-0.79	-0.88	-0.34	0.28
		iii	-0.53	-0.52	-0.27	-0.53	0.05	-0.78	-0.89	-0.43	-0.91	0.10
		iv	0.50	-0.04	0.06	0.22	0.01	0.88	-0.06	0.11	0.43	0.01
		v	-0.06	0.22	0.25	0.46	0.44	-0.09	0.42	0.48	0.85	0.66
2	S30	(4.67)										
		i	0.30	0.30	0.67	0.38	0.10	0.48	0.51	0.97	0.64	0.12
		ii	-0.21	0.38	0.34	-0.14	-0.69	-0.33	0.58	0.68	-0.25	-1.23
		iii	-0.72	0.37	-0.03	0.15	-1.12	-1.20	0.61	-0.06	0.27	-1.72
		iv	-0.25	0.42	0.18	-0.01	0.32	-0.47	0.85	0.36	-0.02	0.41
		v	0.45	0.17	0.26	0.20	-0.15	0.83	0.33	0.49	0.34	-0.23
3	S31	(4.57)										
		i	0.15	0.52	0.14	0.39	0.63	0.20	0.77	0.26	0.72	0.95
		ii	-0.48	-0.24	-0.09	0.14	0.24	-0.75	-0.41	-0.18	0.27	0.43
		iii	-0.91	-0.26	-0.24	0.01	0.11	-1.48	-0.48	-0.35	0.01	0.18
		iv	-0.11	-0.01	0.34	0.39	0.40	-0.20	-0.01	0.51	0.67	0.77
		v	0.34	0.44	0.51	0.14	-0.14	0.59	0.79	0.93	0.26	-0.23
4	S32	(4.64)										
		i	0.50	0.58	0.34	0.55	1.06	0.72	0.93	0.54	0.88	1.62
		ii	-0.60	-0.70	-0.54	-0.26	0.08	-0.95	-1.12	-0.89	-0.45	0.14
		iii	-0.73	-0.84	-0.62	-0.72	-0.29	-1.06	-1.37	-1.03	-1.22	-0.54
		iv	0.11	-0.02	0.05	0.11	-0.03	0.19	-0.04	0.08	0.21	-0.05
		v	0.15	0.08	0.22	0.24	0.53	0.27	0.14	0.41	0.45	0.82
5	S33	(4.58)										
		i	1.23	0.79	0.98	0.51	-0.04	1.85	1.37	1.57	0.87	-0.06
		ii	-0.19	0.31	0.18	-0.27	-0.91	-0.29	0.55	0.34	-0.44	-1.54
		iii	-0.90	-0.02	-0.13	-0.21	-1.25	-1.54	-0.04	-0.22	-0.38	-1.84
		iv	-0.49	0.37	0.23	0.00	-0.03	-0.88	0.74	0.45	0.01	-0.04
		v	0.20	0.17	0.29	0.18	-0.13	0.35	0.33	0.56	0.32	-0.19
6	S34	(4.51)										
		i	0.85	0.63	0.36	0.37	0.51	1.23	1.07	0.68	0.70	0.77
		ii	-0.60	-0.14	-0.29	0.15	-0.20	-0.92	-0.23	-0.58	0.27	-0.34
		iii	-1.28	-0.42	-0.56	-0.26	-0.01	-2.05	-0.77	-0.85	-0.48	-0.01
		iv	-0.46	-0.05	0.20	0.43	0.29	-0.79	-0.09	0.39	0.71	0.56
		v	0.43	0.31	0.24	0.12	0.22	0.73	0.57	0.43	0.23	0.38

Table 3j (contd.): Average Standard Deviations of Returns, Actual Average Returns and their t statistics for the international markets:

S. No.	Samples ($\bar{\sigma}_i$)	$\bar{R}_{i,t}$ (%)					t-stats				
		1	2	3	4	5	1	2	3	4	5
7	S35 (3.89)										
	i	0.76	0.92	0.86	0.75	0.76	1.07	1.73	1.79	1.65	1.77
	ii	0.78	0.62	0.59	0.74	0.37	1.22	1.37	1.23	1.64	0.85
	iii	0.61	0.75	0.45	0.57	0.33	1.03	1.59	0.97	1.29	0.71
	iv	0.57	0.67	0.64	0.31	0.23	0.99	1.53	1.45	0.62	0.46
	v	0.56	0.32	0.51	0.17	0.22	1.06	0.71	1.18	0.34	0.39
8	S36 (3.66)										
	i	0.59	0.81	0.88	0.77	0.99	1.26	1.87	1.94	1.68	1.65
	ii	0.48	0.43	0.61	0.67	0.69	1.09	0.98	1.34	1.39	1.35
	iii	0.64	0.34	0.52	0.49	0.49	1.48	0.74	1.28	1.04	0.90
	iv	0.43	0.44	0.60	0.30	0.65	1.02	0.98	1.33	0.62	1.27
	v	0.27	0.52	0.29	0.36	0.55	0.59	1.18	0.62	0.77	1.08
9	S37 (3.69)										
	i	0.51	0.90	0.84	0.72	1.27	1.10	1.97	1.76	1.55	2.22
	ii	0.27	0.51	0.66	0.85	0.89	0.62	1.12	1.35	1.86	1.67
	iii	0.26	0.53	0.67	0.54	0.63	0.58	1.17	1.56	1.19	1.18
	iv	0.35	0.42	0.66	0.57	0.58	0.73	0.89	1.48	1.21	1.26
	v	0.29	0.38	0.43	0.36	0.39	0.60	0.72	0.91	0.86	0.83
10	S38 (3.95)										
	i	0.94	1.26	1.14	0.92	0.90	1.31	2.25	2.24	1.87	2.02
	ii	0.75	0.67	0.69	0.73	0.41	1.14	1.46	1.44	1.59	0.93
	iii	0.59	0.76	0.49	0.61	0.33	0.96	1.62	1.01	1.34	0.70
	iv	0.54	0.70	0.56	0.35	0.21	0.91	1.59	1.22	0.69	0.44
	v	0.65	0.42	0.50	0.17	0.25	1.20	0.98	1.14	0.34	0.47
11	S39 (3.73)										
	i	0.86	0.93	0.95	0.98	1.19	1.72	2.03	2.05	2.03	1.95
	ii	0.51	0.48	0.65	0.68	0.67	1.13	1.08	1.41	1.40	1.28
	iii	0.65	0.39	0.51	0.54	0.47	1.50	0.82	1.23	1.09	0.83
	iv	0.39	0.45	0.56	0.36	0.55	0.88	1.00	1.24	0.74	1.04
	v	0.27	0.25	0.46	0.44	0.68	0.60	0.55	1.03	0.92	1.36
12	S40 (3.75)										
	i	0.81	1.11	0.97	0.91	1.37	1.64	2.31	1.99	1.91	2.34
	ii	0.28	0.56	0.74	0.80	0.96	0.64	1.22	1.52	1.76	1.77
	iii	0.25	0.58	0.69	0.47	0.71	0.53	1.24	1.57	1.03	1.26
	iv	0.35	0.40	0.63	0.51	0.58	0.73	0.83	1.41	1.04	1.22
	v	0.35	0.40	0.52	0.50	0.32	0.75	0.82	1.12	1.08	0.70

Table 3j (contd.): Average Standard Deviations of Returns, Actual Average Returns and their t statistics for the international markets:

S. No.	Samples ($\bar{\sigma_i}$)	$\bar{R}_{i,t}$ (%)					t-stats				
		1	2	3	4	5	1	2	3	4	5
13	S41 (4.05)										
	i	0.05	0.26	0.31	0.18	0.19	0.10	0.51	0.64	0.37	0.41
	ii	0.31	0.33	0.26	0.21	0.03	0.57	0.64	0.48	0.43	0.05
	iii	0.61	0.64	0.11	0.14	-0.02	1.11	1.17	0.22	0.26	-0.03
	iv	0.67	0.51	0.25	0.12	0.01	1.24	1.00	0.49	0.23	0.01
	v	0.35	0.36	0.21	-0.01	-0.36	0.83	0.75	0.43	-0.01	-0.58
14	S42 (3.98)										
	i	-0.17	0.26	0.36	0.47	0.21	-0.34	0.56	0.78	0.99	0.40
	ii	-0.02	0.15	0.48	0.27	0.24	-0.04	0.29	1.03	0.53	0.43
	iii	0.03	0.21	0.36	0.35	0.40	0.05	0.37	0.74	0.68	0.73
	iv	0.07	0.20	0.39	0.51	0.44	0.12	0.38	0.86	0.96	0.83
	v	0.12	0.14	0.27	-0.06	0.06	0.24	0.31	0.55	-0.12	0.11
15	S43 (4.01)										
	i	-0.27	0.53	0.46	0.57	0.44	-0.54	1.14	0.95	1.11	0.90
	ii	-0.08	0.35	0.19	0.21	0.57	-0.16	0.71	0.37	0.40	1.07
	iii	0.17	0.07	0.72	0.17	0.41	0.33	0.13	1.39	0.30	0.78
	iv	-0.04	0.41	0.44	0.33	0.52	-0.08	0.81	0.85	0.63	0.98
	v	-0.38	0.26	0.17	0.29	0.12	-0.63	0.54	0.35	0.57	0.27
16	S44 (4.16)										
	i	-0.13	-0.01	0.07	-0.05	0.47	-0.26	-0.03	0.14	-0.10	0.98
	ii	0.29	0.20	0.26	0.14	-0.04	0.54	0.38	0.49	0.29	-0.08
	iii	0.48	0.56	0.11	0.04	-0.07	0.83	1.02	0.19	0.08	-0.11
	iv	0.63	0.46	0.28	0.17	-0.25	1.16	0.87	0.53	0.33	-0.39
	v	0.46	0.33	0.14	0.00	-0.16	0.97	0.67	0.27	0.01	-0.25
17	S45 (4.09)										
	i	0.05	0.44	0.37	0.40	-0.16	0.10	0.95	0.86	0.87	-0.31
	ii	-0.10	0.09	0.48	0.18	0.17	-0.18	0.18	1.00	0.35	0.30
	iii	-0.09	0.22	0.31	0.29	0.28	-0.14	0.38	0.62	0.54	0.49
	iv	0.01	0.17	0.34	0.48	0.36	0.02	0.32	0.69	0.87	0.67
	v	0.21	0.15	0.35	-0.02	0.13	0.40	0.31	0.69	-0.04	0.24
18	S46 (4.10)										
	i	-0.12	0.55	0.51	0.41	0.50	-0.23	1.20	1.11	0.85	1.06
	ii	-0.13	0.27	0.12	0.21	0.54	-0.27	0.52	0.22	0.40	1.00
	iii	0.15	-0.02	0.66	0.01	0.33	0.26	-0.04	1.26	0.02	0.60
	iv	-0.26	0.36	0.46	0.40	0.47	-0.46	0.67	0.85	0.75	0.87
	v	-0.21	0.12	0.20	0.35	0.25	-0.34	0.23	0.41	0.70	0.50

Table 3j (contd.): Average Standard Deviations of Returns, Actual Average Returns and their t statistics for the international markets:

S. No.	Samples ($\bar{\sigma_i}$)	$\overline{R}_{i,t}$ (%)					t-stats				
		1	2	3	4	5	1	2	3	4	5
19	S47 (3.66)										
	i	0.17	0.29	0.27	0.35	0.33	0.33	0.64	0.60	0.83	0.78
	ii	0.46	0.54	0.46	0.25	0.13	0.83	1.14	0.97	0.56	0.28
	iii	0.69	0.48	0.39	0.35	0.21	1.31	0.99	0.78	0.72	0.44
	iv	0.74	0.66	0.44	0.33	0.19	1.51	1.41	0.94	0.78	0.38
	v	0.96	0.75	0.52	0.29	0.21	2.13	1.76	1.27	0.69	0.41
20	S48 (3.56)										
	i	0.09	0.44	0.58	0.41	0.21	0.20	1.12	1.49	1.01	0.43
	ii	0.18	0.33	0.47	0.50	0.24	0.38	0.75	1.13	1.13	0.44
	iii	0.25	0.44	0.45	0.35	0.40	0.48	0.98	1.03	0.75	0.73
	iv	0.29	0.50	0.55	0.56	0.54	0.61	1.14	1.34	1.18	1.09
	v	0.51	0.48	0.56	0.63	0.79	1.21	1.21	1.36	1.38	1.51
21	S49 (3.57)										
	i	0.04	0.54	0.39	0.52	0.63	0.10	1.35	0.96	1.23	1.42
	ii	0.16	0.44	0.32	0.36	0.54	0.33	0.99	0.70	0.76	1.14
	iii	0.34	0.46	0.54	0.28	0.42	0.65	0.99	1.13	0.57	0.90
	iv	0.25	0.46	0.59	0.50	0.58	0.50	1.03	1.36	1.09	1.26
	v	0.14	0.51	0.63	0.67	0.75	0.28	1.11	1.50	1.61	1.70
22	S50 (3.82)										
	i	0.21	0.39	0.23	0.37	0.66	0.41	0.81	0.50	0.84	1.45
	ii	0.34	0.49	0.45	0.23	0.11	0.62	0.99	0.94	0.47	0.23
	iii	0.59	0.48	0.39	0.35	0.23	1.07	0.97	0.76	0.70	0.42
	iv	0.74	0.66	0.40	0.33	0.07	1.47	1.40	0.83	0.74	0.14
	v	0.80	0.66	0.50	0.30	0.18	1.79	1.52	1.13	0.68	0.33
23	S51 (3.72)										
	i	0.58	0.61	0.72	0.44	0.11	1.17	1.52	1.78	1.07	0.22
	ii	0.17	0.34	0.47	0.46	0.14	0.32	0.74	1.07	1.00	0.25
	iii	0.22	0.47	0.46	0.36	0.33	0.40	1.00	0.98	0.72	0.58
	iv	0.22	0.48	0.51	0.56	0.52	0.45	1.02	1.17	1.13	0.99
	v	0.49	0.42	0.59	0.53	0.49	1.08	1.00	1.39	1.15	0.96
24	S52 (3.71)										
	i	0.36	0.65	0.51	0.50	0.70	0.73	1.67	1.24	1.17	1.53
	ii	0.08	0.44	0.28	0.30	0.56	0.16	0.95	0.60	0.61	1.13
	iii	0.32	0.46	0.58	0.23	0.37	0.58	0.94	1.17	0.44	0.75
	iv	0.15	0.45	0.57	0.52	0.55	0.28	0.94	1.26	1.08	1.17
	v	0.27	0.46	0.56	0.56	0.55	0.53	0.97	1.29	1.27	1.28

Table 4a: Correlations and Sum of Squared Errors (SSE) between Actual and Estimated Average Returns for the US market

S. No.	Samples	FF5F			RFM		
		Correlation (%)	t-stats	SSE	Correlation (%)	t-stats	SSE
1	S1	34.03	0.63	6.25E-05	99.40	15.72	6.04E-08
2	S2	-66.64	-1.55	9.45E-05	100.00	256.64	2.01E-09
3	S3	1.49	0.03	5.36E-05	99.58	18.76	5.59E-09
4	S4	94.54	5.03	4.57E-05	99.96	60.69	8.35E-09
5	S5	6.44	0.11	6.68E-05	99.11	12.91	8.01E-08
6	S6	89.06	3.39	9.84E-05	99.85	31.30	1.21E-07
7	S7	91.93	4.05	9.36E-05	99.98	83.93	8.58E-08
8	S8	77.51	2.12	9.15E-05	99.94	51.99	8.77E-08
9	S9	21.28	0.62	1.29E-04	99.34	24.52	1.48E-07
10	S10	-61.47	-2.20	2.14E-04	99.96	106.86	1.61E-08
11	S11	4.29	0.12	1.27E-04	99.78	42.13	4.73E-08
12	S12	76.55	3.37	1.10E-04	99.84	50.53	8.28E-08
13	S13	6.44	0.18	1.40E-04	99.23	22.72	1.51E-07
14	S14	73.81	3.09	2.04E-04	99.77	41.75	2.73E-07
15	S15	90.92	6.17	1.82E-04	99.94	83.93	1.82E-07
16	S16	72.91	3.01	1.83E-04	99.86	53.08	1.61E-07
17	S17	6.45	0.31	5.98E-04	99.80	75.63	3.31E-07
18	S18	34.44	1.76	4.17E-04	99.76	68.39	2.58E-07
19	S19	62.97	3.89	4.49E-04	99.66	57.75	4.81E-07
20	S20	-39.20	-2.04	6.50E-04	99.77	70.87	3.39E-07
21	S21	-12.11	-0.58	7.30E-04	99.65	56.80	6.04E-07
22	S22	45.93	2.48	3.61E-04	99.82	79.93	2.62E-07
23	S23	14.76	0.72	6.06E-04	99.73	65.16	4.22E-07
24	S24	60.53	3.65	4.34E-04	99.77	69.87	3.58E-07
25	S25	67.03	4.33	4.55E-04	99.62	54.87	5.39E-07
26	S26	43.17	2.30	5.39E-04	99.83	81.93	5.50E-07
27	S27	14.04	0.68	7.40E-04	99.69	60.77	9.94E-07
28	S28	72.21	5.01	5.33E-04	99.86	89.26	4.37E-07

Table 4b: Correlations and Sum of Squared Errors (SSE) between Actual and Estimated Average Returns for the international markets

S. No.	Samples	FF5F			RFM		
		Correlation	t-stats	SSE	Correlation	t-stats	SSE
1	S29	46.19	2.50	2.76E-04	99.61	54.28	2.55E-06
2	S30	45.83	2.47	4.06E-04	99.75	67.92	2.52E-06
3	S31	55.77	3.22	3.05E-04	99.50	47.62	3.35E-06
4	S32	75.86	5.58	5.68E-04	99.66	58.34	5.87E-06
5	S33	78.36	6.05	7.09E-04	99.83	81.02	4.25E-06
6	S34	80.78	6.57	4.97E-04	99.74	66.09	3.97E-06
7	S35	76.41	5.68	7.29E-04	99.37	42.66	2.37E-06
8	S36	74.49	5.35	6.84E-04	99.57	51.74	1.40E-06
9	S37	80.05	6.41	7.97E-04	99.55	50.43	2.37E-06
10	S38	79.33	6.25	9.50E-04	99.61	54.31	2.59E-06
11	S39	80.59	6.53	8.35E-04	99.64	56.42	1.70E-06
12	S40	83.84	7.38	9.80E-04	99.67	58.99	2.26E-06
13	S41	87.45	8.65	2.17E-04	99.56	50.92	1.57E-06
14	S42	59.43	3.54	1.79E-04	99.32	41.00	1.60E-06
15	S43	69.69	4.66	3.00E-04	99.62	54.91	1.84E-06
16	S44	81.92	6.85	1.84E-04	99.57	51.50	1.38E-06
17	S45	43.39	2.31	1.61E-04	99.17	36.98	1.59E-06
18	S46	68.76	4.54	2.70E-04	99.61	54.13	1.67E-06
19	S47	90.07	9.94	4.41E-04	99.80	75.16	9.79E-07
20	S48	72.55	5.06	4.29E-04	99.58	52.15	8.41E-07
21	S49	78.08	5.99	4.59E-04	99.81	76.99	7.04E-07
22	S50	80.70	6.55	4.13E-04	99.73	65.49	8.39E-07
23	S51	35.50	1.82	4.28E-04	99.66	57.67	6.26E-07
24	S52	52.05	2.92	4.46E-04	99.74	66.74	5.80E-07

Figure 1: Charts of the Actual and the Estimated Average Returns for Value-Weighted Univariate Quintiles for the US market

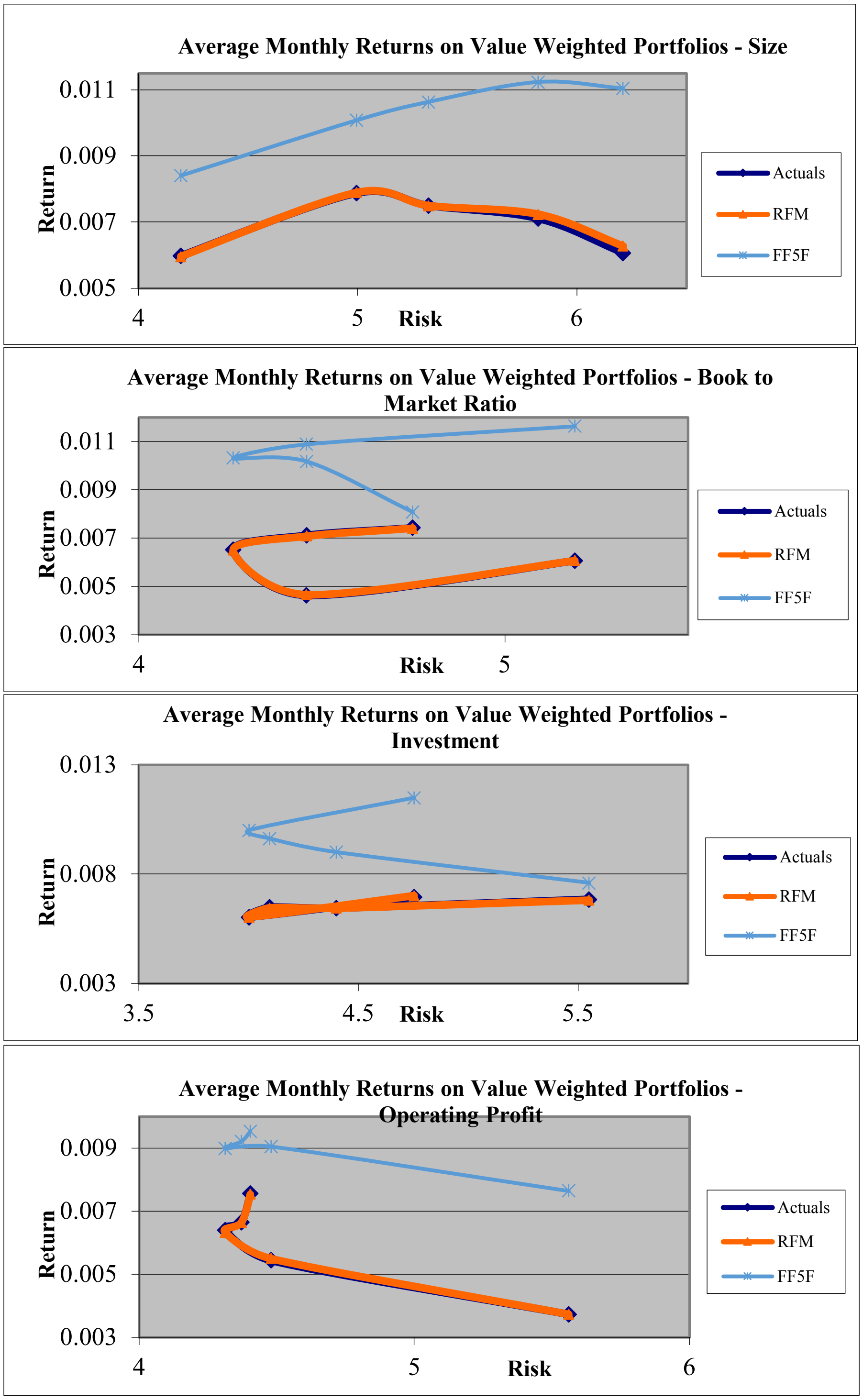

Figure 2: Charts of the Actual and the Estimated Average Returns for Equal-Weighted Univariate Quintiles for the US market

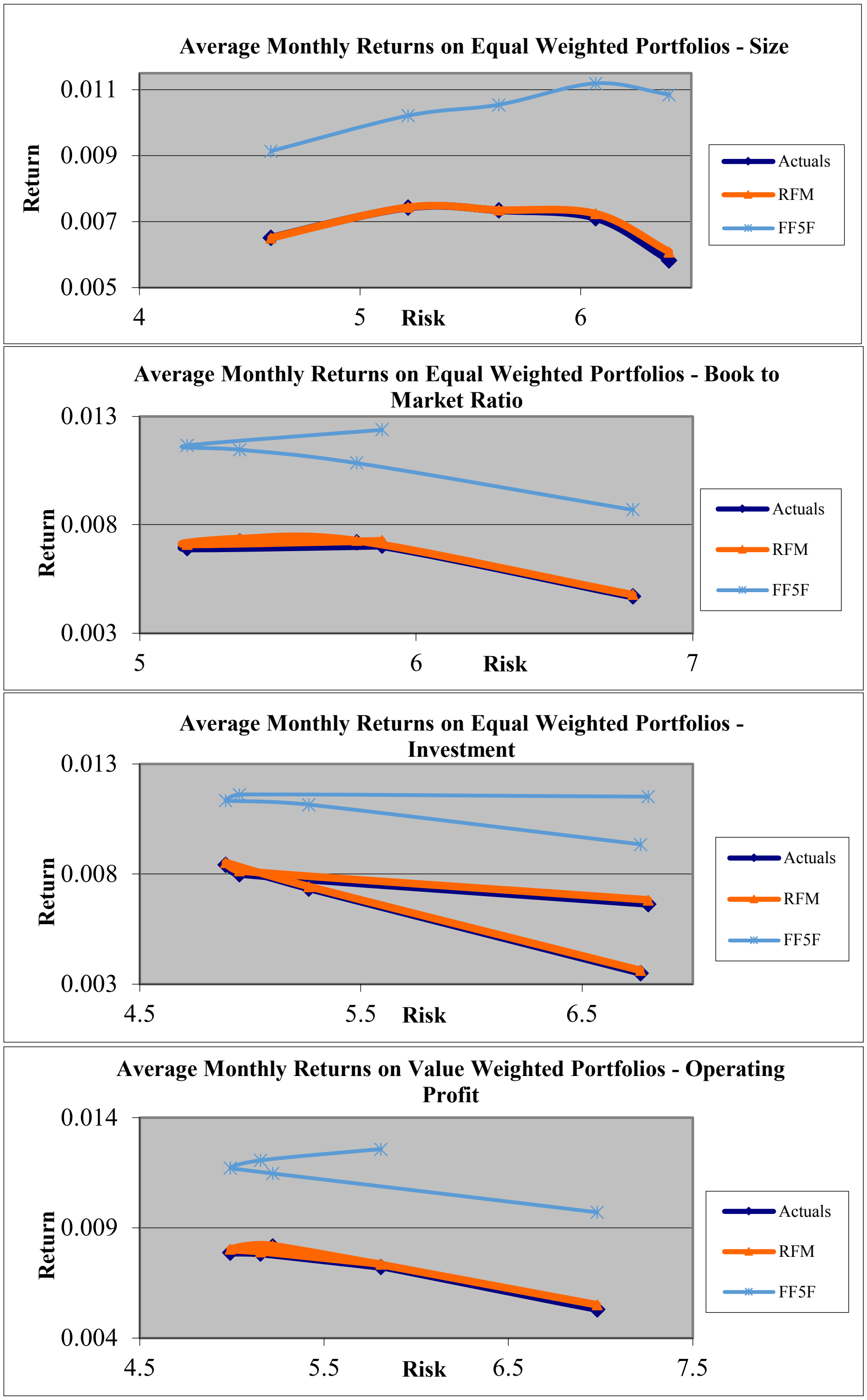

Table 5: Slope coefficients and t-stats of the RFM Equation (8) for Univariate Quintiles for the US market:

$$\boldsymbol{Var_1: \quad Var_{i,t} = \theta_{i,m}\, Var_{m,t} + e_{i,t}}$$

S. No.	Samples	1	2	3	4	5	1	2	3	4	5
		$\theta_{i,m}$					t-stats				
1	S1	1.82	1.66	1.41	1.28	0.91	54.38	89.62	114.71	154.29	239.03
2	S2	1.14	1.02	0.93	1.03	1.32	124.46	127.31	110.48	57.01	55.10
3	S3	1.13	0.80	0.87	1.03	1.54	83.98	111.88	137.81	162.25	88.60
4	S4	1.58	1.03	0.97	0.99	0.99	72.44	116.64	136.99	175.45	111.95
5	S5	1.94	1.87	1.64	1.44	1.14	58.43	81.63	83.31	107.65	122.39
6	S6	2.30	1.70	1.48	1.34	1.74	48.09	88.75	81.07	69.42	55.04
7	S7	2.23	1.25	1.25	1.44	2.29	56.33	75.26	81.71	85.38	53.75
8	S8	2.36	1.40	1.29	1.39	1.72	48.56	76.13	72.87	78.78	70.59

Table 6: Slope coefficients and t-stats of the RFM Equation (9) for Univariate Quintiles for the US market:

$$Var_2: Var_{i,t} = \theta_{i,m}\, Var_{m,t} + \theta_{i,s}\, Var_{SMB,t} + \theta_{i,h}\, Var_{HML,t} + \theta_{i,r}\, Var_{RMW,t} + \theta_{i,c}\, Var_{CMA,t} + e_{i,t}$$

S. No.	Portfolios	Slopes					t-stats				
		$\theta_{i,m}$	$\theta_{i,s}$	$\theta_{i,h}$	$\theta_{i,r}$	$\theta_{i,c}$	$\theta_{i,m}$	$\theta_{i,s}$	$\theta_{i,h}$	$\theta_{i,r}$	$\theta_{i,c}$
1	S1										
	i	1.18	1.79	0.00	0.03	-0.09	36.74	24.66	-0.03	0.36	-0.63
	ii	1.36	1.02	-0.19	-0.50	0.57	51.86	17.28	-2.44	-7.06	4.72
	iii	1.27	0.48	-0.17	-0.37	0.61	63.96	10.62	-2.93	-6.82	6.59
	iv	1.20	0.25	0.01	-0.07	-0.04	87.04	7.95	0.28	-1.80	-0.57
	v	0.91	-0.03	0.03	0.05	-0.03	133.70	-1.76	1.27	2.52	-0.93
2	S2										
	i	1.06	0.13	-0.11	-0.11	0.58	68.27	3.59	-2.29	-2.63	8.05
	ii	1.08	0.14	-0.32	-0.05	0.06	81.50	4.65	-8.03	-1.51	0.99
	iii	0.95	0.13	0.10	-0.16	-0.47	67.32	4.20	2.31	-4.23	-7.18
	iv	0.99	-0.02	1.07	-0.52	-1.33	34.88	-0.26	12.63	-6.81	-10.17
	v	1.25	0.37	0.88	-0.80	-1.32	31.57	4.09	7.43	-7.40	-7.20
3	S3										
	i	1.28	0.05	-0.16	-0.04	-0.66	58.00	0.94	-2.48	-0.66	-6.40
	ii	0.85	0.24	-0.14	-0.16	-0.35	77.55	9.72	-4.24	-5.37	-6.98
	iii	0.93	0.13	-0.02	-0.23	-0.39	112.53	6.70	-0.85	-10.32	-10.05
	iv	1.07	0.10	-0.12	-0.19	-0.03	107.40	4.36	-3.88	-6.81	-0.59
	v	1.15	0.20	0.08	0.02	1.82	60.61	4.56	1.32	0.38	20.71
4	S4										
	i	1.45	-0.73	0.30	1.18	0.35	51.70	-11.53	3.61	15.56	2.67
	ii	1.08	-0.06	0.01	0.00	-0.19	69.26	-1.76	0.21	-0.06	-2.62
	iii	0.94	-0.15	0.18	0.14	-0.05	79.10	-5.50	4.94	4.24	-0.91
	iv	0.88	0.13	0.13	-0.16	0.28	100.96	6.82	4.83	-6.57	6.95
	v	1.02	0.36	-0.26	-0.31	-0.04	75.72	11.89	-6.50	-8.40	-0.72

Table 6 (contd.): Slope coefficients and t-stats of the RFM Equation (9) for Univariate Quintiles for the US market:

$$Var_2\text{: } Var_{i,t} = \theta_{i,m}\, Var_{m,t} + \theta_{i,s}\, Var_{SMB,t} + \theta_{i,h}\, Var_{HML,t} + \theta_{i,r}\, Var_{RMW,t} + \theta_{i,c}\, Var_{CMA,t} + e_{i,t}$$

S. No.	Portfolios	Slopes					t-stats				
		$\theta_{i,m}$	$\theta_{i,s}$	$\theta_{i,h}$	$\theta_{i,r}$	$\theta_{i,c}$	$\theta_{i,m}$	$\theta_{i,s}$	$\theta_{i,h}$	$\theta_{i,r}$	$\theta_{i,c}$
5	S5										
	i	1.20	1.11	0.48	-0.66	1.84	26.64	10.82	3.54	-5.34	8.81
	ii	1.50	0.25	0.58	-0.56	1.16	40.53	3.02	5.18	-5.53	6.74
	iii	1.38	-0.28	0.60	-0.20	1.23	45.01	-4.08	6.52	-2.36	8.66
	iv	1.31	-0.24	0.45	0.01	0.33	59.81	-4.86	6.87	0.18	3.22
	v	1.07	-0.37	0.60	0.06	-0.05	81.07	-12.52	15.26	1.81	-0.79
6	S6										
	i	1.42	-0.47	0.54	1.20	3.97	27.94	-4.09	3.56	8.74	16.95
	ii	1.44	0.56	0.15	-0.55	0.66	45.22	7.79	1.62	-6.38	4.48
	iii	1.36	0.57	0.31	-0.88	-0.15	45.17	8.45	3.47	-10.73	-1.06
	iv	1.12	0.86	0.54	-1.12	-0.40	37.64	12.80	6.09	-13.81	-2.87
	v	1.30	1.33	1.37	-1.64	-1.14	26.81	12.12	9.47	-12.52	-5.09
7	S7										
	i	1.49	0.82	1.03	0.25	0.23	28.89	7.03	6.64	1.81	0.96
	ii	1.14	0.50	0.63	-0.85	-0.75	43.37	8.37	8.07	-11.86	-6.16
	iii	1.14	0.60	0.52	-0.87	-0.74	49.37	11.48	7.51	-13.81	-6.94
	iv	1.26	0.67	0.42	-0.99	-0.05	47.06	11.12	5.29	-13.59	-0.42
	v	1.41	0.03	0.63	-0.01	4.11	26.71	0.27	4.01	-0.05	16.86
8	S8										
	i	1.39	0.46	0.71	0.90	2.35	25.55	3.71	4.32	6.09	9.31
	ii	1.21	0.74	0.61	-1.16	-0.33	42.70	11.60	7.15	-15.09	-2.50
	iii	1.18	0.67	0.64	-1.22	-0.67	46.37	11.76	8.43	-17.66	-5.69
	iv	1.28	0.71	0.57	-1.29	-0.56	52.89	12.88	7.83	-19.54	-5.00
	v	1.51	1.01	0.77	-1.69	-0.53	42.38	12.58	7.25	-17.53	-3.22

Table 7a: Correlations and Sum of Squared Errors (SSE) between Actual and Estimated Average Variances for the US market:

S. No.	Samples	Var_1: Equation (8)		Var_2: Equation (9)		Equation with smaller SSE	Difference between the SSEs (%)
		Correlation (%)	SSE	Correlation (%)	SSE		
1	S1	99.93	1.93E-03	99.92	7.42E-05	Var_2	96.15
2	S2	99.37	3.03E-04	98.94	3.04E-04	Var_1	-0.24
3	S3	99.56	2.57E-04	99.13	3.76E-04	Var_1	-46.40
4	S4	99.99	5.83E-05	99.88	6.53E-05	Var_1	-12.06
5	S5	99.79	1.01E-03	99.92	8.53E-05	Var_2	91.52
6	S6	99.89	5.11E-04	99.87	4.35E-04	Var_2	14.92
7	S7	99.91	9.72E-04	99.82	4.80E-04	Var_2	50.62
8	S8	99.93	9.18E-04	99.99	2.08E-04	Var_2	77.37
9	S9	99.80	4.14E-11	99.88	1.99E-12	Var_2	95.19
10	S10	99.38	6.40E-12	99.68	6.25E-12	Var_2	2.27
11	S11	99.51	1.05E-11	99.13	8.20E-12	Var_2	22.19
12	S12	99.56	1.14E-11	99.91	3.16E-12	Var_2	72.38
13	S13	99.57	1.98E-11	99.92	1.23E-12	Var_2	93.75
14	S14	99.83	1.16E-11	99.87	8.70E-12	Var_2	24.84
15	S15	99.89	2.72E-11	99.86	8.50E-12	Var_2	68.76
16	S16	99.92	2.20E-11	99.98	3.90E-12	Var_2	82.25
17	S17	99.66	1.66E-10	99.78	1.82E-11	Var_2	89.05
18	S18	99.75	1.28E-10	99.63	2.27E-11	Var_2	82.17
19	S19	99.65	1.33E-10	99.81	1.34E-11	Var_2	89.92
20	S20	96.12	9.06E-11	98.05	4.98E-11	Var_2	45.09
21	S21	99.67	2.78E-10	99.59	1.21E-10	Var_2	56.46
22	S22	98.82	6.70E-11	98.90	3.84E-11	Var_2	42.59
23	S23	99.62	9.08E-11	99.83	1.84E-11	Var_2	79.76
24	S24	99.65	6.44E-11	99.57	2.97E-11	Var_2	53.89
25	S25	99.59	6.01E-11	99.76	9.58E-12	Var_2	84.07
26	S26	99.70	9.62E-11	99.63	2.22E-11	Var_2	76.90
27	S27	99.73	1.79E-10	99.83	1.44E-11	Var_2	91.96
28	S28	99.82	6.85E-11	99.86	1.15E-11	Var_2	83.22

Table 7b: Correlations and Sum of Squared Errors (SSE) between Actual and Estimated Average Variances for the international markets:

S. No.	Samples	Var_1: Equation (8)		Var_2: Equation (9)		Equation with smaller SSE	Difference between the SSEs (%)
		Correlation (%)	SSE	Correlation (%)	SSE		
1	S29	99.31	1.86E-03	99.96	1.30E-04	Var_2	93.00
2	S30	98.87	2.39E-03	99.86	2.66E-04	Var_2	88.85
3	S31	97.99	2.71E-03	99.88	1.63E-04	Var_2	93.99
4	S32	91.48	5.72E-03	99.56	2.94E-04	Var_2	94.86
5	S33	92.40	4.32E-03	99.42	3.70E-04	Var_2	91.44
6	S34	93.76	3.70E-03	99.62	2.44E-04	Var_2	93.40
7	S35	96.48	8.66E-03	99.91	1.63E-04	Var_2	98.12
8	S36	92.89	4.38E-03	99.79	1.09E-04	Var_2	97.51
9	S37	89.89	4.68E-03	99.77	1.04E-04	Var_2	97.77
10	S38	96.66	8.22E-03	99.92	1.89E-04	Var_2	97.70
11	S39	90.80	4.58E-03	99.78	1.25E-04	Var_2	97.26
12	S40	86.34	4.77E-03	99.72	1.21E-04	Var_2	97.46
13	S41	99.19	3.85E-04	99.81	3.95E-05	Var_2	89.75
14	S42	99.22	3.22E-04	99.69	6.08E-05	Var_2	81.14
15	S43	99.18	3.04E-04	99.73	5.91E-05	Var_2	80.55
16	S44	99.56	5.36E-04	99.89	7.68E-05	Var_2	85.65
17	S45	98.93	4.77E-04	99.66	8.89E-05	Var_2	81.37
18	S46	99.39	4.33E-04	99.87	7.58E-05	Var_2	82.49
19	S47	98.92	5.92E-04	99.75	4.50E-05	Var_2	92.40
20	S48	99.26	4.02E-04	99.85	4.47E-05	Var_2	88.88
21	S49	98.96	3.56E-04	99.71	4.33E-05	Var_2	87.83
22	S50	96.75	1.28E-03	99.82	4.36E-05	Var_2	96.58
23	S51	97.63	1.10E-03	99.87	3.92E-05	Var_2	96.45
24	S52	97.48	8.32E-04	99.84	3.14E-05	Var_2	96.22