Rational Functions: A New Approach to Asset Pricing

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Introduction

Modern Portfolio Theory, as we know it, was originated by Markowitz (1952).

This theory defined '**Risk**' for the first time, as the Variance of Returns.

It also outlined the modus operandi for '**efficient**' portfolio selection through the following twin objectives:

a) Maximizing Portfolio Return, i.e. Mean of the Portfolio Returns

b) Minimizing Portfolio Risk, i.e. Variance of the Portfolio Returns

This concept of forming 'Mean-Variance' efficient Portfolios, when linked with the Market Environment, further gave rise to the Asset Pricing Models, the first one being the Capital Asset Pricing Model (CAPM) formulated by Sharpe (1964) and Lintner (1965).

Existing Asset Pricing Models

The Modern Portfolio Theory paved the way for various Asset Pricing Models like the CAPM, the APT, the ICAPM, the Fama French three Factor Model (FF3F), the Fama French five Factor Model (FF5F) etc.

However, all these models are based on one fundamental assumption and that is – **All Stock and Asset Returns add linearly in a Portfolio**.

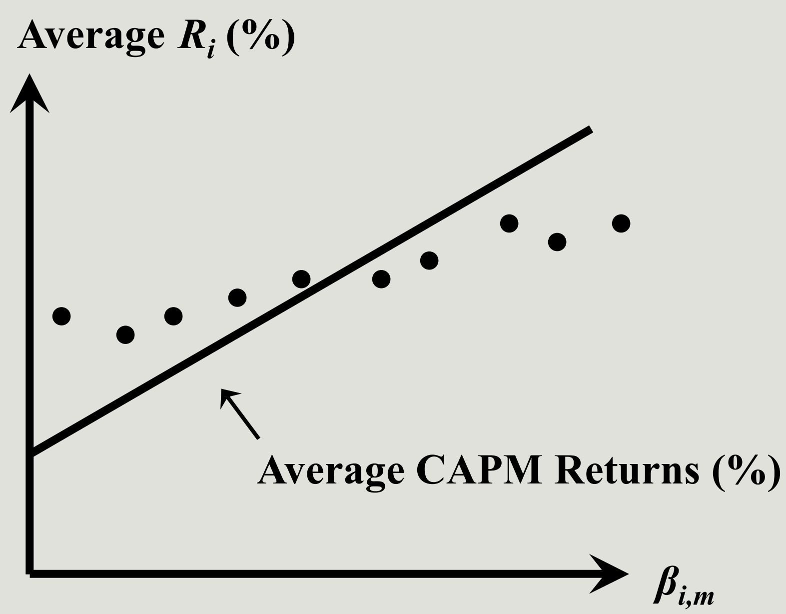
Hence, these models obtain the Portfolio and Index Returns by averaging the constituent Stock Returns and studying a possible linear relationship between these Stock and Index Returns. In the 3-Factor model, two additional factors synthesized from accounting and market parameters are also considered. Similarly in the 5-Factor model, two further accounting parameters are considered.

Of all the existing Asset Pricing Models, the CAPM and the FF5F are the most popularly used.

CAPM and the Joint Hypothesis Problem

CAPM Equation: $E(R_i) = R_f + \beta_{i,m} [E(R_m) - R_f], \quad i = 1, 2, \ldots, N.$

FF5F Model: $E(R_{i,t}) - R_{f,t} = \beta_{i,m} [E(R_{m,t}) - R_{f,t}) + \beta_{i,s} E(SMB_t) + \beta_{i,h} E(HML_t) + \beta_{i,v} E(RMW_t) + \beta_{i,c} E(CMA_t)$



Plot of Actual and CAPM Average Returns against $\beta_{i,m}$ (Fama and French 2004).

Various empirical studies reported typical discrepancies between the Actual Returns and the CAPM Returns as shown in the Figure. These discrepancies are called as Anomalies.

As a result, Fama (1970) stated the **Joint Hypothesis Problem** which attributes these anomalies to either:

i) a flaw in the Model; and/or

ii) inefficiencies in the Market

Motivation for this Study

The existing Asset Pricing theories have so far modeled **only** the Asset Returns.

However, the basic tenets of Economics and Finance have always spoken of a **Demand-Supply** framework based on Price and Quantity that drives a conventional market.

The stock market is no different from the conventional commodities market since here also the sellers are constrained by their **finite holdings** of the stocks while the buyers are constrained by their **budgets**. These constraints shape the forces of supply and demand.

The main motivation behind this study is to fulfill the need to re-visit the basics to understand the market behavior properly and thereby develop a more accurate Asset Pricing model by identifying the relevant variables.

Re-visiting the Basics

The basic economic fundamentals of a market rest on the laws of **Supply and Demand**. However, these laws define the price-setting mechanism of a single good in an insulated market environment.

We combine these laws of Supply and Demand with the real life situation of multiple similar assets trading in a free market environment reflected in the behavior of a **Market Index**. Thus we consider the **prices and volumes** of the Stocks and the Market Index.

We further account for the possible effects of **time trends**, **other market factors and past performance** on the asset prices.

This gives us a linear polynomial defining the price of a publicly traded asset as:

$$p_{i,t}=\alpha_i+\beta_{i1}\{(p_{m,t}/p_{m,t-1})p_{i,t-1}\}+\beta_{i2}exp(v_{i,t})+\beta_{i3}ln(v_{i,t})+\beta_{i4}exp(v_{m,t})+\beta_{i5}ln(v_{m,t})+\beta_{i6}p_{i,t-1}+\beta_{i7}ln(t_t)+\beta_{i8}MF_{i,t}$$

Empirical Validation of the New Theory

The model described in the previous slide represents a new concept.

This conceptual model was further refined through empirical tests and the following models were found to give the best results for two different types of returns:

a) Average Returns across increasing risk are computed from:

$$p_{i,t} = \beta_i \, [\{(p_{m,t} / p_{m,t-1}) \, p_{i,t-1}\}] + e_{it}$$

b) Continuous Returns across both increasing risk and time are computed from:

$$R_{i,t} - R_{f,t} = \beta_{i,m}(R_{m,t} - R_{f,t}) + \beta_{i,s}SMB_t + \beta_{i,h}HML_t + \beta_{i,r}RMW_t + \beta_{i,c}CMA_t + \beta_{i,v}(V_{m,t}) + \beta_{i,o}(t_t)^2 + \beta_{i,l}(R_{i,t-1}) + e_{it}$$

The first equation is for average returns while the second equation is for continuous returns and is a combination of the FF5F and RFM.

For the latter case, the continuous asset returns are found to be 'approximately' linear and hence they are modeled directly through linear regression.

The Intercept for the Average Returns

The assumption of a risk free rate of lending and borrowing that was used for CAPM and FF5F has been tested using a risk-free component of asset prices denoted as ‘α_i’.

However, we have also tested a zero-intercept version of the model where the intercept is assumed to be zero because that would be the lowest price that could be payable for an asset since the intercept cannot be negative as no asset has negative price in a supply-demand framework.

Thus we have tested two versions of the new model using both with as well as without the intercept. The results reported in this presentation are that of the model without intercept, i.e. $\alpha_i = 0$.

It should be mentioned that the empirical values of the average risk-free rate of return (i.e. R_f) are negligible and has been taken to be zero over the past few years as given in Prof. Kenneth French’s website.

Methodology for Empirical Tests

Hereby presented are the results of empirical tests conducted in order to demonstrate the practical authenticity of the new theory discussed previously. The data used is from the stock markets of **USA, Australia and India** during the years **2003-2013**.

For the tests, 21 samples were constructed using the **constituent stocks** forming the **DJIA, B400, S&P500, Fama French Portfolios, ASX50 & ASX Midcap50 and finally BSE Sensex**. Of these, 13 samples used **monthly** data, while the remaining 8 samples used **daily** data at different times within the larger time period.

The market indices used are the **DJIA and S&P500 for US market, ASX All Ordinaries for the Australian market and BSE Sensex for the Indian market**.

The portfolio returns were computed as ratios of two consecutive cross-sectional average prices as per the new RFM theory.

Samples Studied for Empirical Testing

S. No.	Portfolios	Market	Data from	to	Type of Returns	Number of time intervals	Sorting Factor(s)	Market Proxy	Names
1	30 components of DJIA as on April 30, 2013	USA	30-May-03	30-Apr-13	Monthly	120	Returns Variance	DJIA	S1
2	30 components of DJIA as on April 30, 2013	USA	30-May-03	30-Apr-13	Monthly	120	Returns Variance	S&P 500	S2
3	30 components of DJIA as on April 30, 2013	USA	30-Jun-05	30-Apr-13	Monthly	95	Returns Variance	DJIA	S3
4	30 components of DJIA as on April 30, 2013	USA	30-Jun-05	30-Apr-13	Monthly	95	Returns Variance	S&P 500	S4
5	30 components of DJIA as on April 30, 2013	USA	12-Dec-12	30-Apr-13	Daily	95	Returns Variance	DJIA	S5
6	30 components of DJIA as on April 30, 2013	USA	12-Dec-12	30-Apr-13	Daily	95	Returns Variance	S&P 500	S6
7	396 components of B400 as on August 1, 2013	USA	12-Dec-12	30-Apr-13	Daily	95	Returns Variance	S&P 500	S7
8	500 components of S&P 500 as on August 1, 2013	USA	12-Dec-12	30-Apr-13	Daily	95	Returns Variance	S&P 500	S8
9	30 components of DJIA as on April 30, 2013	USA	30-May-03	30-Apr-13	Monthly	120	Idiosyncratic Volatility	S&P 500	S9
10	396 components of B400 as on August 1, 2013	USA	12-Dec-12	30-Apr-13	Daily	95	Idiosyncratic Volatility	S&P 500	S10
11	500 components of S&P 500 as on August 1, 2013	USA	12-Dec-12	30-Apr-13	Daily	95	Idiosyncratic Volatility	S&P 500	S11
12	Fama-French 5 Portfolios of All USA stocks	USA	30-May-03	30-Apr-13	Monthly	120	Industry	S&P 500	S12
13	Fama-French 6 Portfolios of All USA stocks	USA	30-May-03	30-Apr-13	Monthly	120	Size & Investment	S&P 500	S13
14	Fama-French 6 Portfolios of All USA stocks	USA	30-May-03	30-Apr-13	Monthly	120	Size & Long term reversals	S&P 500	S14
15	Fama-French 6 Portfolios of All USA stocks	USA	30-May-03	30-Apr-13	Monthly	120	Size and Momentum	S&P 500	S15
16	Fama-French 6 Portfolios of All USA stocks	USA	30-May-03	30-Apr-13	Monthly	120	Size and Operating profits	S&P 500	S16
17	Fama-French 6 Portfolios of All USA stocks	USA	30-May-03	30-Apr-13	Monthly	120	Size & Short term reversals	S&P 500	S17
18	Fama-French 6 Portfolios of All USA stocks	USA	30-May-03	30-Apr-13	Monthly	120	Size and BE/ME ratio	S&P 500	S18
19	100 components of S&P ASX 50 and S&P ASX Mid-Cap 50 as on May 15, 2013	Australia	20-May-13	30-Sep-13	Daily	95	Returns Variance	ASX All Ordinaries	S19
20	100 components of S&P ASX 50 and S&P ASX Mid-Cap 50 as on May 15, 2013	Australia	12-Apr-13	30-Sep-13	Daily	120	Returns Variance	ASX All Ordinaries	S20
21	30 components of BSE Sensex as on January 1, 2005	India	31-Jan-02	30-Nov-09	Monthly	95	Returns Variance	BSE Sensex	S21

Results: Values of $Actual_1$ Average Returns (calculated as ratios of average prices):

Names	P1	P2	P3	P4	P5	P-full or P6	P1	P2	P3	P4	P5	P-full or P6
	$\beta_{i,m\text{-CAPM}}$						$Actual_1$ Average Return					
S1	0.74	0.77	1.07	1.27	1.67	1.10	0.32%	0.82%	0.47%	0.41%	-0.36%	0.34%
S2	0.64	0.69	0.99	1.19	1.55	1.01	0.32%	0.82%	0.47%	0.41%	-0.36%	0.34%
S3	0.73	0.76	1.09	1.29	1.72	1.10	0.24%	0.95%	0.54%	0.21%	-0.32%	0.34%
S4	0.63	0.68	1.00	1.19	1.58	1.00	0.24%	0.95%	0.54%	0.21%	-0.32%	0.34%
S5	0.82	1.04	1.02	1.14	1.17	1.04	0.13%	0.17%	0.18%	0.18%	0.14%	0.16%
S6	0.62	0.91	0.86	0.96	1.01	0.87	0.13%	0.17%	0.18%	0.18%	0.14%	0.16%
S7	0.91	1.05	1.10	1.21	1.29	1.11	0.13%	0.16%	0.14%	0.13%	0.15%	0.14%
S8	0.84	0.96	1.11	1.23	1.25	1.08	0.17%	0.14%	0.14%	0.15%	0.13%	0.14%
S9	0.81	0.85	0.84	1.06	1.48	1.01	0.40%	0.85%	0.33%	0.07%	0.24%	0.37%
S10	0.95	1.06	1.13	1.17	1.25	1.11	0.14%	0.14%	0.17%	0.12%	0.15%	0.14%
S11	0.91	1.00	1.09	1.23	1.16	1.08	0.15%	0.14%	0.17%	0.14%	0.12%	0.15%
S12	0.82	1.02	1.10	0.68	1.27	0.97	0.86%	0.80%	0.63%	0.75%	0.16%	0.63%
S13	1.37	1.18	1.29	0.97	0.96	1.05	0.78%	0.77%	0.50%	0.62%	0.62%	0.53%
S14	1.51	1.18	1.30	1.16	0.95	0.98	0.64%	0.80%	0.75%	0.47%	0.63%	0.60%
S15	1.61	1.24	1.25	1.40	0.97	0.98	0.72%	0.78%	0.69%	0.34%	0.74%	0.64%
S16	1.36	1.18	1.25	1.17	1.07	0.88	0.46%	0.79%	0.84%	0.38%	0.55%	0.68%
S17	1.57	1.27	1.23	1.36	0.96	0.96	0.63%	0.72%	0.56%	0.13%	0.74%	0.66%
S18	1.26	1.21	1.34	0.90	1.07	1.22	0.54%	0.74%	0.75%	0.64%	0.50%	0.60%
S19	0.76	0.94	0.98	1.15	1.44	1.04	0.05%	0.12%	0.07%	0.15%	0.08%	0.09%
S20	0.79	0.96	1.00	1.19	1.42	1.06	0.01%	0.04%	0.00%	0.04%	0.04%	0.03%
S21	1.00	1.00	1.01	1.04	1.06	1.02	1.17%	1.66%	1.84%	1.70%	1.05%	1.50%

Results: Correlations between $Actual_1$ Average Returns and Estimated Average Returns across P1 to P-full:

Sample Portfolios	CAPM		FF3F		FF5F		RF_{1a} (with intercept)		RF_{1b} (without intercept)	
	Correlation	t-stats	Correlation	t-stats	Correlation	t-stats	Correlation	t-stats	Correlation	t-stats
S1	-81.74%	-2.46	-81.68%	-2.45	-55.51%	-1.16	99.90%	39.00	99.88%	34.68
S2	-79.49%	-2.27	-81.55%	-2.44	-60.03%	-1.30	99.87%	34.14	99.86%	32.73
S3	-79.36%	-2.26	-68.78%	-1.64	77.48%	2.12	99.76%	24.82	99.75%	24.49
S4	-77.19%	-2.10	-60.82%	-1.33	83.23%	2.60	99.72%	23.23	99.73%	23.72
S5	46.82%	0.92	44.88%	0.87	72.15%	1.80	97.84%	8.19	98.23%	9.09
S6	52.44%	1.07	50.13%	1.00	72.49%	1.82	98.09%	8.74	98.77%	10.94
S7	33.23%	0.61	30.46%	0.55	49.76%	0.99	97.91%	8.35	98.27%	9.20
S8	-75.70%	-2.01	-65.33%	-1.49	3.23%	0.06	99.17%	13.32	99.12%	12.96
S9	-46.40%	-0.91	-43.23%	-0.83	58.83%	1.26	99.58%	18.85	99.66%	21.07
S10	7.28%	0.13	-0.01%	-1.81E-04	37.54%	0.70	99.07%	12.61	98.82%	11.19
S11	-31.71%	-0.58	-5.48%	-0.09	63.98%	1.44	99.32%	14.82	99.42%	16.08
S12	-76.81%	-2.08	-81.77%	-2.46	-17.56%	-0.31	99.82%	28.66	99.86%	32.70
S13	34.61%	0.74	54.86%	1.14	86.78%	3.03	98.94%	11.83	99.13%	15.08
S14	23.42%	0.48	39.60%	0.75	65.04%	1.48	99.24%	14.01	99.32%	17.12
S15	-22.79%	-0.47	19.88%	0.35	89.06%	3.39	99.58%	18.76	99.66%	24.30
S16	-15.68%	-0.32	27.98%	0.50	93.81%	4.69	99.59%	19.06	99.62%	22.97
S17	-36.48%	-0.78	0.40%	0.01	65.85%	1.52	99.68%	21.73	99.74%	27.75
S18	35.51%	0.76	55.35%	1.15	74.98%	1.96	98.68%	10.57	98.89%	13.30
S19	27.37%	0.49	28.12%	0.51			99.91%	40.56	99.92%	43.87
S20	65.35%	1.50	75.89%	2.02			99.94%	52.10	99.95%	53.85
S21	39.22%	0.74	17.13%	0.30			99.99%	150.53	99.99%	127.82

Results: Correlations between $Actual_2$ Average Returns (calculated as averages of time series of continuous returns) and Estimated Average Returns across P1 to P-full:

Sample Portfolios	CAPM		FF3F		FF5F		RF_{1a} (with intercept)		RF_{1b} (without intercept)	
	Correlation	t-stats	Correlation	t-stats	Correlation	t-stats	Correlation	t-stats	Correlation	t-stats
S1	-81.86%	-2.47	-82.50%	-2.53	-57.90%	-1.23	99.64%	20.45	99.60%	19.25
S2	-79.65%	-2.28	-82.52%	-2.53	-62.44%	-1.38	99.59%	19.08	99.57%	18.63
S3	-79.85%	-2.30	-69.26%	-1.66	77.72%	2.14	99.68%	21.49	99.67%	21.25
S4	-77.69%	-2.14	-61.39%	-1.35	83.47%	2.62	99.64%	20.27	99.65%	20.65
S5	43.02%	0.83	41.67%	0.79	75.61%	2.00	98.78%	11.01	98.89%	11.53
S6	48.78%	0.97	47.15%	0.93	75.42%	1.99	99.01%	12.22	99.32%	14.81
S7	34.54%	0.64	32.29%	0.59	48.46%	0.96	97.90%	8.32	98.15%	8.88
S8	-73.59%	-1.88	-62.69%	-1.39	7.16%	0.12	98.74%	10.79	98.65%	10.45
S9	-53.91%	-1.11	-49.62%	-0.99	56.43%	1.18	99.58%	18.92	99.51%	17.42
S10	8.35%	0.15	1.32%	0.02	35.92%	0.67	98.69%	10.59	98.38%	9.51
S11	-28.20%	-0.51	-1.78%	-0.03	65.80%	1.51	99.40%	15.71	99.45%	16.43
S12	-61.01%	-1.33	-70.00%	-1.70	-10.29%	-0.18	97.96%	8.44	97.81%	8.14
S13	43.03%	0.95	62.87%	1.40	89.39%	3.45	99.00%	12.17	98.91%	13.42
S14	17.83%	0.36	31.04%	0.57	55.67%	1.16	98.00%	8.52	97.98%	9.81
S15	-27.06%	-0.56	16.07%	0.28	88.51%	3.29	98.74%	10.83	98.97%	13.81
S16	-12.61%	-0.25	30.93%	0.56	93.58%	4.60	99.61%	19.51	99.64%	23.52
S17	-34.59%	-0.74	3.23%	0.06	68.76%	1.64	99.77%	25.22	99.81%	32.04
S18	43.75%	0.97	63.49%	1.42	80.35%	2.34	96.25%	6.14	96.61%	7.48
S19	25.07%	0.45	25.78%	0.46			99.85%	31.58	99.86%	32.44
S20	64.65%	1.47	76.18%	2.04			99.93%	46.91	99.94%	48.35
S21	42.11%	0.80	40.19%	0.76			95.96%	5.91	96.24%	6.14

Results: Sum of Squared Errors of Average Returns (SSEA) between $Actual_1$ Average Returns and Estimated Average Returns across P1 to P-full:

Sample Portfolios	CAPM	FF3F	FF5F	RF_{1a} (with intercept)	RF_{1b} (without intercept)	Improvement of RF_{1b} over CAPM	Improvement of RF_{1b} over FF3F	Improvement of RF_{1b} over FF5F
S1	1.01E-04	1.27E-04	9.22E-05	3.86E-07	4.17E-07	99.59%	99.67%	99.55%
S2	9.59E-05	1.19E-04	9.53E-05	3.84E-07	4.26E-07	99.56%	99.64%	99.55%
S3	1.20E-04	1.15E-04	5.40E-05	5.63E-07	6.03E-07	99.50%	99.48%	98.88%
S4	1.08E-04	1.05E-04	5.14E-05	5.98E-07	6.18E-07	99.43%	99.41%	98.80%
S5	2.31E-07	2.24E-07	1.61E-07	2.34E-08	1.80E-08	92.20%	91.96%	88.85%
S6	9.67E-07	7.77E-07	4.96E-07	3.05E-08	2.37E-08	97.55%	96.95%	95.23%
S7	2.19E-07	1.78E-07	7.41E-08	9.80E-09	8.32E-09	96.19%	95.34%	88.77%
S8	5.35E-07	3.37E-07	9.99E-08	2.20E-09	2.44E-09	99.54%	99.28%	97.56%
S9	4.16E-05	4.84E-05	2.60E-05	4.33E-07	4.14E-07	99.00%	99.14%	98.41%
S10	2.95E-07	2.92E-07	1.70E-07	1.37E-08	1.13E-08	96.16%	96.12%	93.33%
S11	3.36E-07	2.01E-07	7.88E-08	2.76E-09	2.79E-09	99.17%	98.62%	96.46%
S12	8.86E-05	9.39E-05	8.37E-05	1.24E-07	9.44E-08	99.89%	99.90%	99.89%
S13	5.29E-05	2.91E-05	2.97E-05	3.64E-07	3.96E-07	99.25%	98.64%	98.67%
S14	5.46E-05	3.11E-05	3.50E-05	3.88E-07	4.58E-07	99.16%	98.52%	98.69%
S15	6.01E-05	3.52E-05	3.94E-05	2.54E-07	3.75E-07	99.38%	98.93%	99.05%
S16	5.78E-05	3.45E-05	2.90E-05	4.47E-07	4.83E-07	99.16%	98.60%	98.33%
S17	5.33E-05	3.95E-05	3.54E-05	2.99E-07	3.47E-07	99.35%	99.12%	99.02%
S18	4.71E-05	2.18E-05	2.61E-05	4.24E-07	4.48E-07	99.05%	97.95%	98.28%
S19	7.37E-07	8.28E-07		1.00E-08	1.36E-08	98.16%	98.36%	
S20	2.18E-07	2.73E-07		5.89E-09	6.62E-09	96.97%	97.57%	
S21	9.64E-05	1.12E-04		5.86E-07	3.84E-07	99.60%	99.66%	

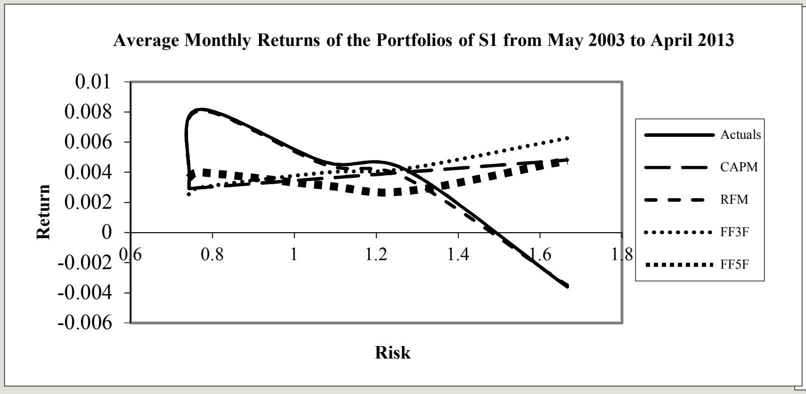
Results: Sum of Squared Errors of Average Returns (SSEA) between Actual$_2$ Average Returns and Estimated Average Returns across P1 to P-full:

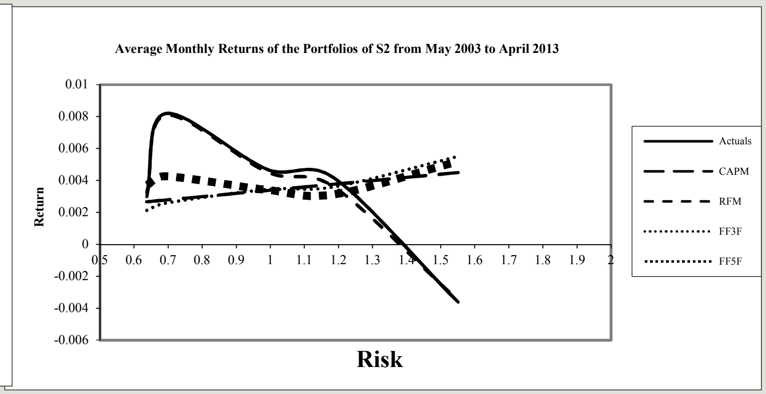
Sample Portfolios	CAPM	FF3F	FF5F	RF_{1a} (with intercept)	RF_{1b} (without intercept)	Improvement of RF_{1b} over CAPM	Improvement of RF_{1b} over FF3F	Improvement of RF_{1b} over FF5F
S1	8.66E-05	1.13E-04	7.92E-05	9.86E-07	1.05E-06	98.78%	99.07%	98.67%
S2	8.08E-05	1.03E-04	8.33E-05	1.13E-06	1.11E-06	98.63%	98.92%	98.67%
S3	1.02E-04	9.71E-05	4.09E-05	1.77E-06	1.65E-06	98.38%	98.30%	95.96%
S4	8.70E-05	8.34E-05	3.85E-05	1.92E-06	1.73E-06	98.01%	97.92%	95.51%
S5	2.48E-07	2.37E-07	1.64E-07	1.48E-08	1.21E-08	95.12%	94.90%	92.64%
S6	1.05E-06	8.45E-07	5.33E-07	2.03E-08	1.52E-08	98.55%	98.20%	97.15%
S7	1.85E-07	2.33E-07	1.07E-07	2.69E-08	2.46E-08	86.72%	89.45%	77.04%
S8	5.12E-07	3.39E-07	1.00E-07	7.29E-09	8.16E-09	98.40%	97.59%	91.86%
S9	3.38E-05	4.08E-05	2.12E-05	1.05E-06	1.07E-06	96.82%	97.37%	94.94%
S10	2.59E-07	3.43E-07	2.00E-07	3.10E-08	2.76E-08	89.31%	91.93%	86.19%
S11	3.21E-07	2.11E-07	8.48E-08	7.83E-09	8.43E-09	97.38%	96.01%	90.06%
S12	6.68E-05	7.19E-05	6.30E-05	3.13E-06	3.14E-06	95.30%	95.63%	95.01%
S13	3.55E-05	1.72E-05	1.78E-05	3.23E-06	3.46E-06	90.25%	79.90%	80.54%
S14	4.07E-05	2.33E-05	2.55E-05	2.73E-06	3.00E-06	92.62%	87.13%	88.24%
S15	4.45E-05	2.57E-05	2.62E-05	2.70E-06	3.17E-06	92.88%	87.69%	87.93%
S16	4.63E-05	2.67E-05	2.01E-05	2.20E-06	2.35E-06	94.92%	91.20%	88.33%
S17	3.97E-05	3.01E-05	2.38E-05	2.34E-06	2.61E-06	93.43%	91.33%	89.03%
S18	3.68E-05	1.48E-05	1.80E-05	2.18E-06	2.30E-06	93.76%	84.48%	87.25%
S19	7.72E-07	8.72E-07		7.25E-09	1.00E-08	98.71%	98.85%	
S20	1.97E-07	2.48E-07		4.49E-09	4.88E-09	97.52%	98.03%	
S21	8.41E-05	3.52E-05		4.67E-05	4.72E-05	43.90%	-34.13%	

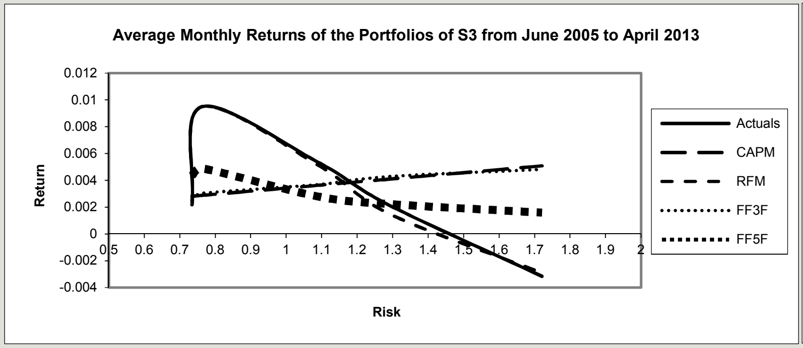
Average Sum of Squared Errors (SSEs) between the Actual and the Estimated Continuous Returns

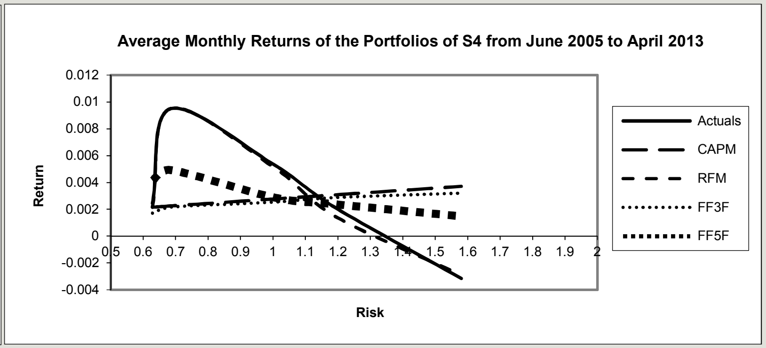
Portfolios	CAPM	FF3F	FF5F	RF_{2a}	RF_{2b}	$(SSE_{CAPM}-SSE_{FF5F})/SSE_{CAPM}$	$(SSE_{CAPM}-SSE_{RF2b})/SSE_{CAPM}$	$(SSE_{FF5F}-SSE_{RF2b})/SSE_{FF5F}$
S1	5.66E-02	5.00E-02	4.77E-02	4.74E-02	4.58E-02	15.76%	19.03%	3.88%
S2	5.87E-02	5.30E-02	5.11E-02	5.01E-02	4.88E-02	12.94%	16.73%	4.36%
S3	4.88E-02	3.99E-02	3.61E-02	3.70E-02	3.43E-02	26.19%	29.68%	4.73%
S4	5.18E-02	4.30E-02	3.97E-02	3.98E-02	3.78E-02	23.38%	27.02%	4.75%
S5	1.23E-03	1.12E-03	1.05E-03	1.06E-03	9.78E-04	15.04%	20.61%	6.56%
S6	1.26E-03	1.17E-03	1.10E-03	1.12E-03	1.04E-03	13.25%	17.68%	5.10%
S7	1.01E-03	4.59E-04	4.24E-04	4.24E-04	3.89E-04	57.94%	61.46%	8.35%
S8	4.69E-04	3.13E-04	2.86E-04	2.88E-04	2.62E-04	39.04%	44.12%	8.32%
S9	6.08E-02	5.71E-02	5.49E-02	5.41E-02	5.25E-02	9.63%	13.64%	4.43%
S10	9.98E-04	4.62E-04	4.28E-04	4.24E-04	3.90E-04	57.18%	60.95%	8.81%
S11	4.24E-04	2.85E-04	2.67E-04	2.65E-04	2.47E-04	37.00%	41.73%	7.51%
S12	3.71E-02	3.01E-02	2.86E-02	2.85E-02	2.72E-02	22.93%	26.73%	4.94%
S13	3.55E-02	7.58E-03	4.46E-03	6.79E-03	4.06E-03	87.43%	88.54%	8.86%
S14	4.93E-02	1.82E-02	1.55E-02	1.71E-02	1.43E-02	68.51%	71.01%	7.93%
S15	7.46E-02	4.51E-02	4.23E-02	4.17E-02	3.99E-02	43.22%	46.52%	5.81%
S16	3.61E-02	8.24E-03	4.68E-03	7.67E-03	4.28E-03	87.05%	88.16%	8.61%
S17	5.23E-02	2.59E-02	2.41E-02	2.43E-02	2.29E-02	53.97%	56.12%	4.69%
S18	4.36E-02	5.74E-03	4.89E-03	5.52E-03	4.61E-03	88.80%	89.42%	5.62%
S19	1.46E-03	1.34E-03		1.29E-03				
S20	2.37E-03	1.99E-03		1.97E-03				
S21	1.18E-01	9.80E-02		9.45E-02				
	Paired t-test					H_0: $(SSE_{CAPM} - SSE_{FF5F})/ SSE_{CAPM} \leq 0$	H_0: $(SSE_{CAPM} - SSE_{RF2b})/ SSE_{CAPM} \leq 0$	H_0: $(SSE_{FF5F} - SSE_{RF2b})/ SSE_{FF5F} \leq 0$
	t-statistic					6.35	7.13	14.39
	p-value					3.6E-06	8.4E-07	3.0E-11

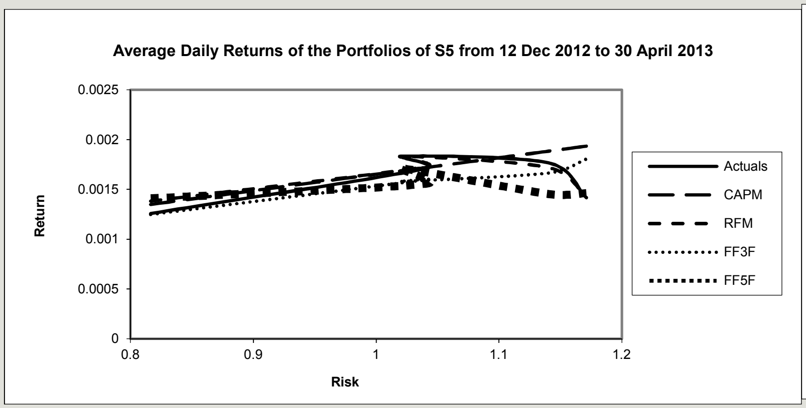
Charts of Average Returns – Actual$_1$ and Estimated

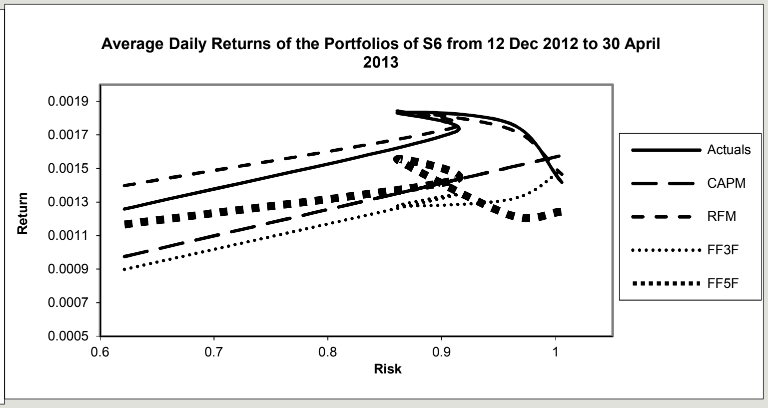


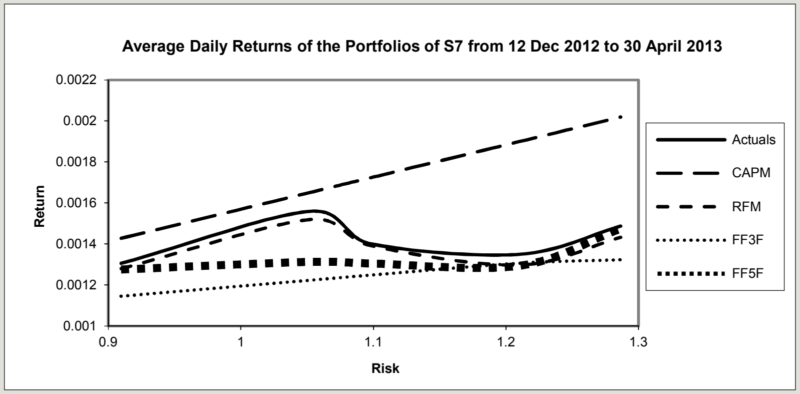


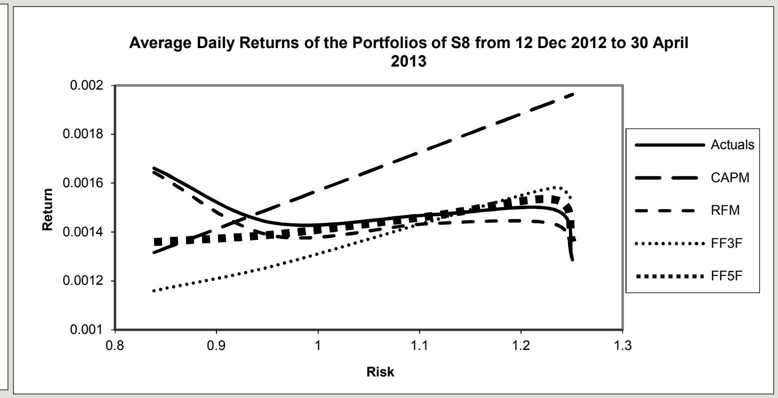


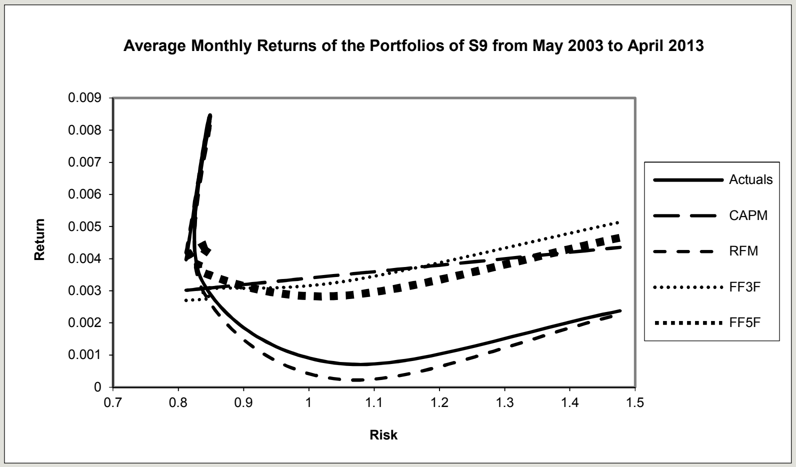


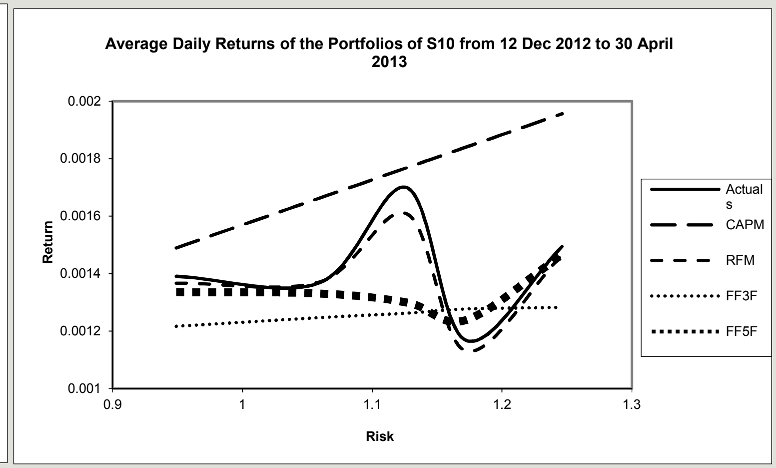


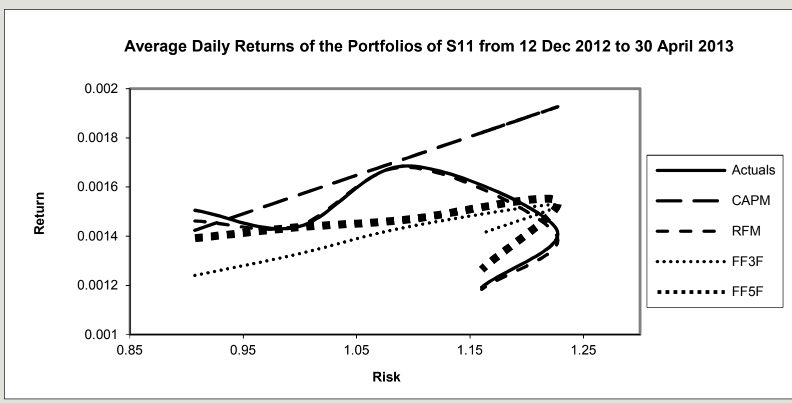


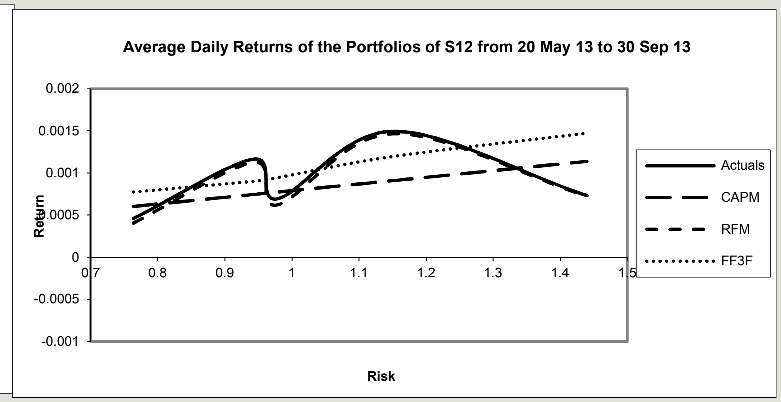

Average Daily Returns of the Portfolios of S7 from 12 Dec 2012 to 30 April 2013
Return
0.0022
0.002
0.0018
0.0016
0.0014
0.0012
0.001
0.9
1
1.1
1.2
1.3
Risk
Actuals
CAPM
RFM
FF3F
FF5F

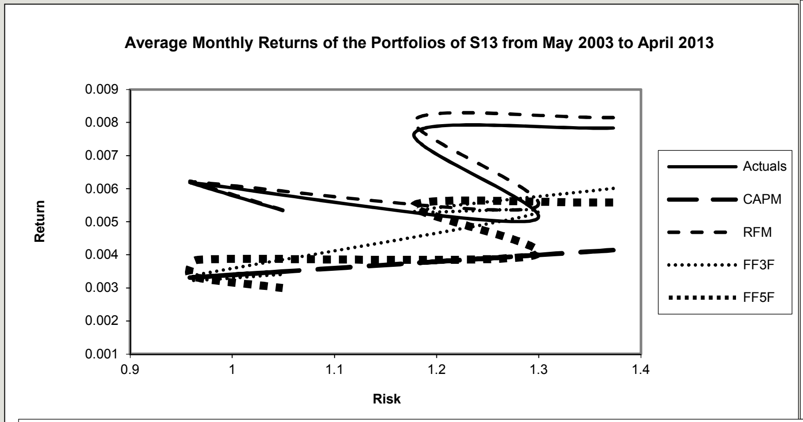

Average Daily Returns of the Portfolios of S8 from 12 Dec 2012 to 30 April 2013
Return
0.002
0.0018
0.0016
0.0014
0.0012
0.001
0.8
0.9
1
1.1
1.2
1.3
Risk
Actuals
CAPM
RFM
FF3F
FF5F

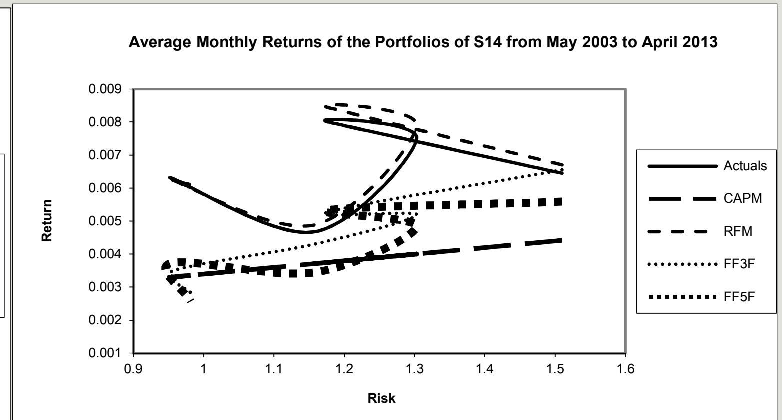

Average Monthly Returns of the Portfolios of S9 from May 2003 to April 2013
Return
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4
1.5
Risk
Actuals
CAPM
RFM
FF3F
FF5F

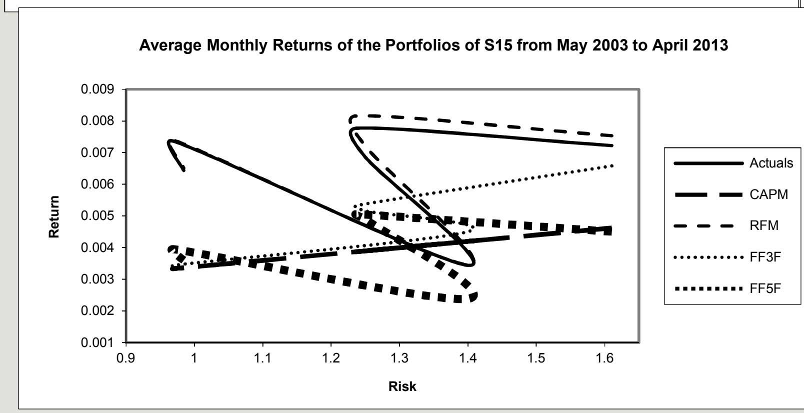

Average Daily Returns of the Portfolios of S10 from 12 Dec 2012 to 30 April 2013
Return
0.002
0.0018
0.0016
0.0014
0.0012
0.001
0.9
1
1.1
1.2
1.3
Risk
Actual
s
CAPM
RFM
FF3F
FF5F

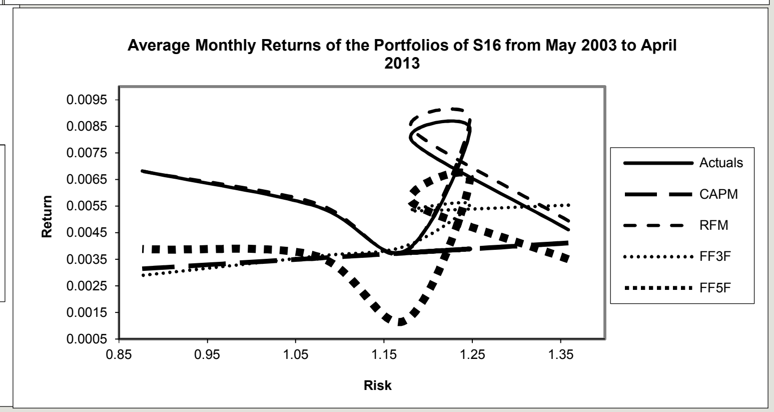

Average Daily Returns of the Portfolios of S11 from 12 Dec 2012 to 30 April 2013
Return
0.002
0.0018
0.0016
0.0014
0.0012
0.001
0.85
0.95
1.05
1.15
1.25
Risk
Actuals
CAPM
RFM
FF3F
FF5F

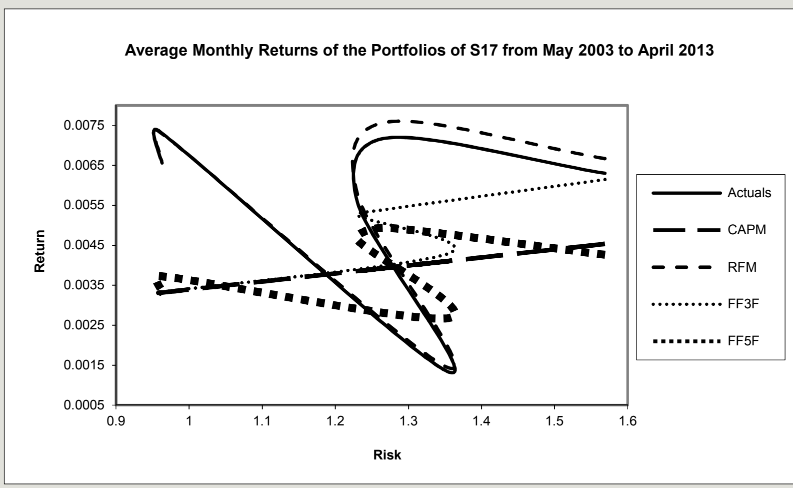

Average Daily Returns of the Portfolios of S12 from 20 May 13 to 30 Sep 13
Return
0.002
0.0015
0.001
0.0005
0
-0.0005
-0.001
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4
1.5
Risk
Actuals
CAPM
RFM
FF3F

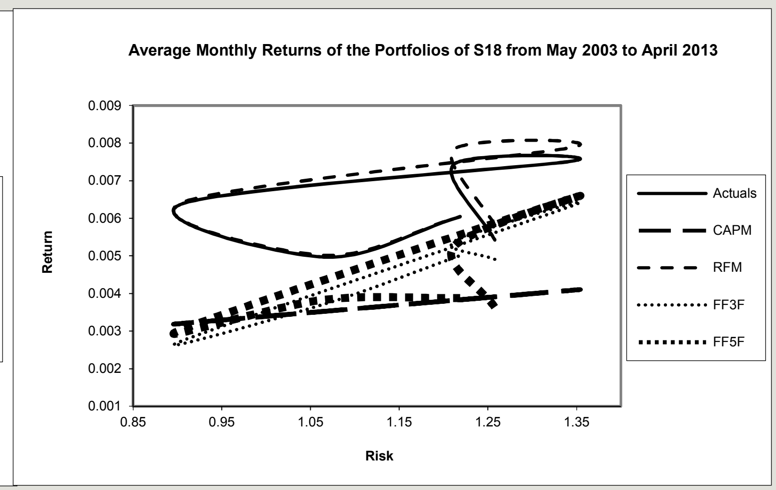

Average Monthly Returns of the Portfolios of S13 from May 2003 to April 2013
Return
Risk
Actuals
CAPM
RFM
FF3F
FF5F

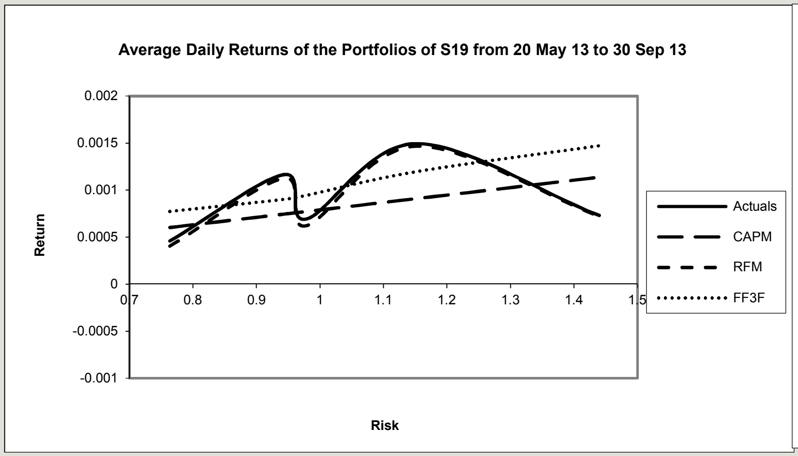

Average Monthly Returns of the Portfolios of S14 from May 2003 to April 2013
Return
Risk
Actuals
CAPM
RFM
FF3F
FF5F

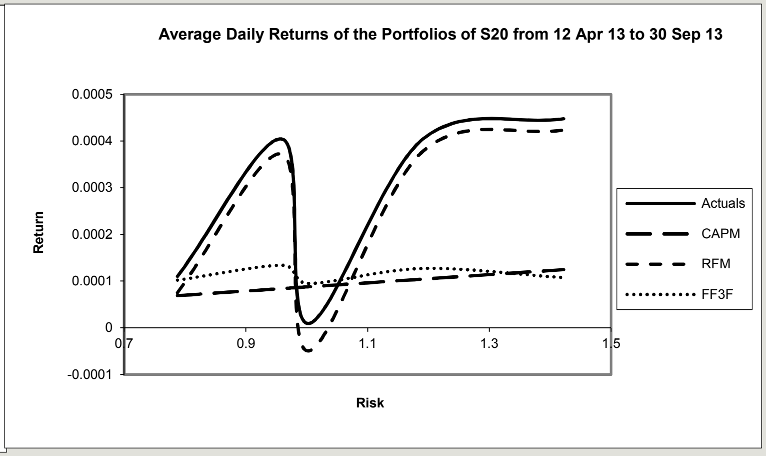

Average Monthly Returns of the Portfolios of S15 from May 2003 to April 2013
Return
Risk
Actuals
CAPM
RFM
FF3F
FF5F

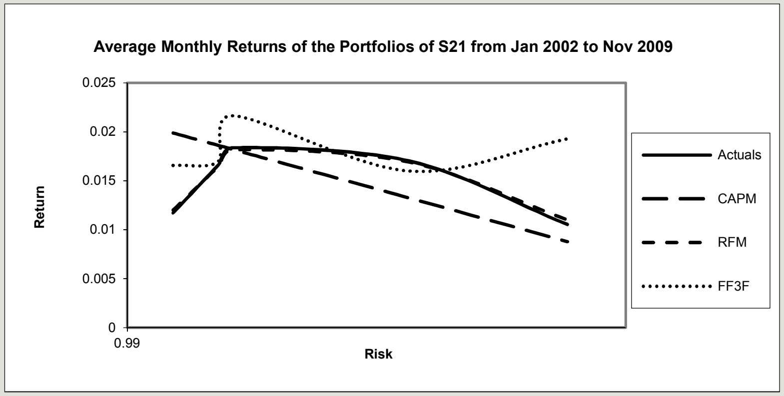

Average Monthly Returns of the Portfolios of S16 from May 2003 to April 2013
Return
Risk
Actuals
CAPM
RFM
FF3F
FF5F


Average Monthly Returns of the Portfolios of S17 from May 2003 to April 2013
Return
Risk
Actuals
CAPM
RFM
FF3F
FF5F


Average Monthly Returns of the Portfolios of S18 from May 2003 to April 2013
Return
Risk
Actuals
CAPM
RFM
FF3F
FF5F


Average Daily Returns of the Portfolios of S19 from 20 May 13 to 30 Sep 13
Return
Risk
Actuals
CAPM
RFM
FF3F


Average Daily Returns of the Portfolios of S20 from 12 Apr 13 to 30 Sep 13
Return
Risk
Actuals
CAPM
RFM
FF3F


Average Monthly Returns of the Portfolios of S21 from Jan 2002 to Nov 2009
Return
Risk
Actuals
CAPM
RFM
FF3F

Empirical Observations

1. The RFM gives **more accurate results** than CAPM or FF5F models for **Average Returns** across increasing Risk.

2. **Sorting** is not an issue for estimating average returns using RFM.

3. For the RF_1 models, the **values of $\boldsymbol{\beta_i}$** are all positive and very **close to 1.00**. This maybe because the change in the asset price $p_{i,t}$ is nearly equal to the change in market price $p_{m,t}$.

4. The *t*-statistics of the **intercepts $\boldsymbol{\alpha_i}$** for RF_{1a} model are all **insignificant** and the intercepts themselves are roughly within a range of -1.2% to +1.9% of the average asset prices.

5. For **Continuous Returns** measured across both Time and Risk, the Combined FF5F-RFM model gives consistently **better results** than the CAPM and the FF5F Models.

Further Considerations

It can be further empirically shown that average volumes can also be estimated using the RFM theory. For this we define change in volume $V_{i,t}$ as

$$V_{i,t} = ln(v_{i,t} / v_{i,t-1})$$

Then, average $V_{i,t}$ can be estimated using the following RF model

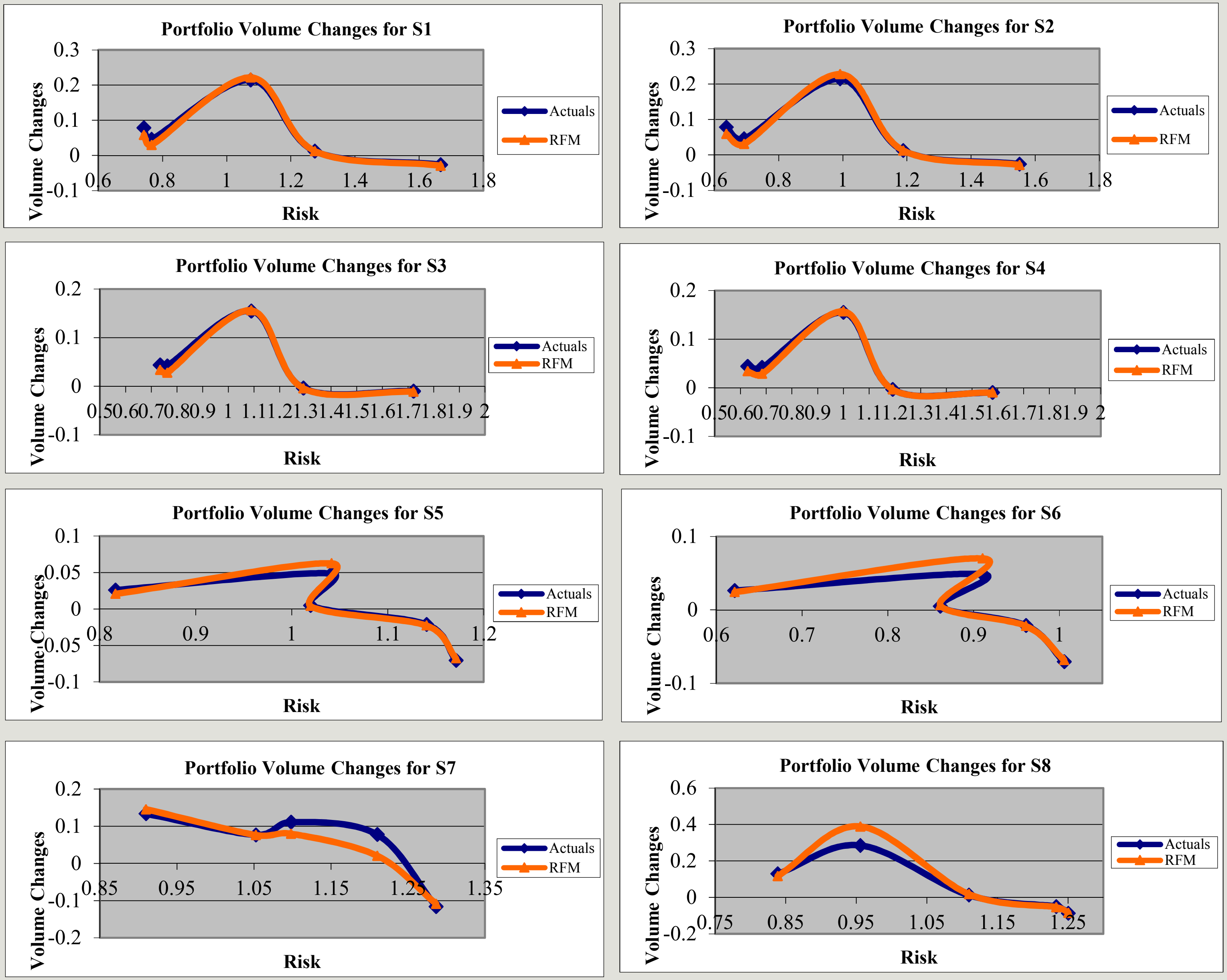
$$v_{i,t} = \gamma_i [\{(v_{m,t} / v_{m,t-1}) v_{i,t-1}\}] + e_{it}$$

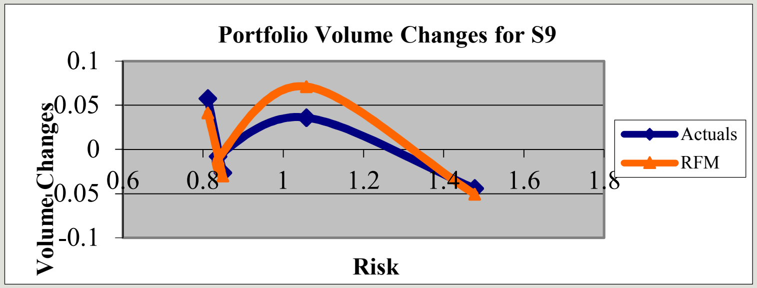
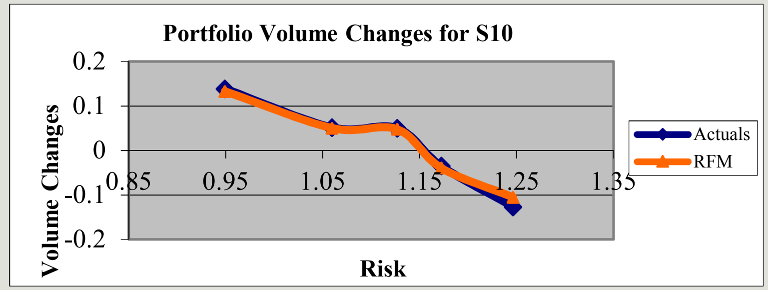
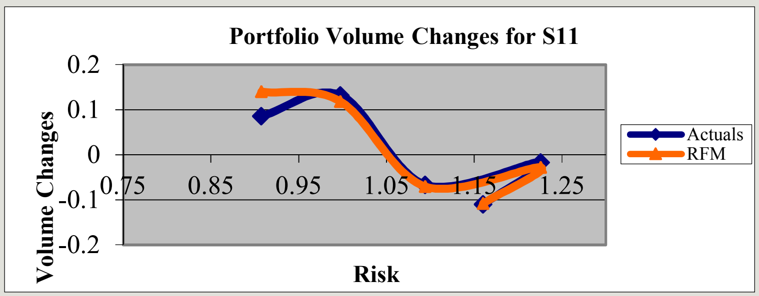
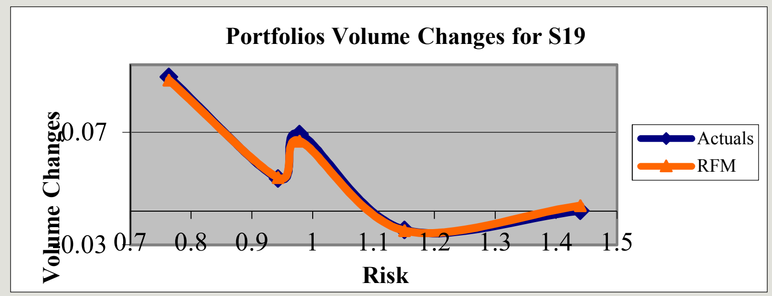
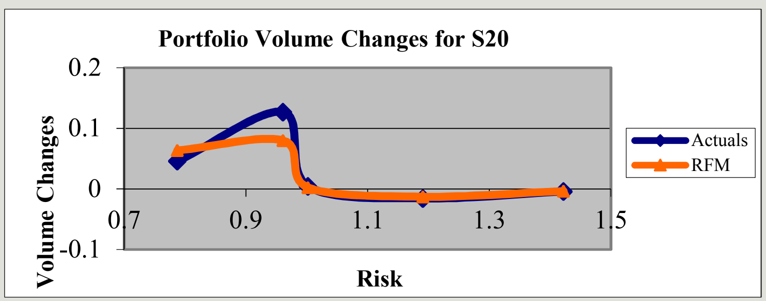
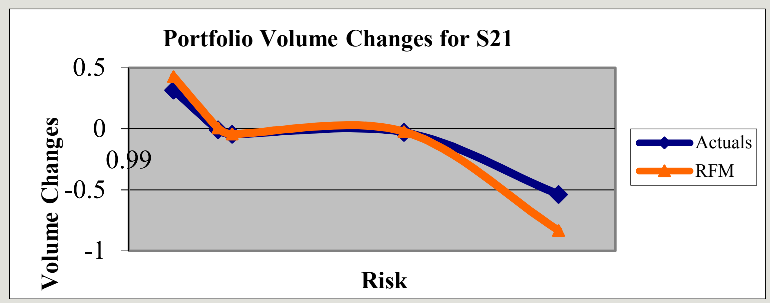
Change in asset volumes $V_{i,t}$ shows the change in liquidity of the asset and indicates the degree of realizability of the returns of that asset.

Results: Correlations and SSE between Actual Average $V_{i,t}$ and Estimated Average $V_{i,t}$ across P1 to P-full for the samples for which volume data were collected

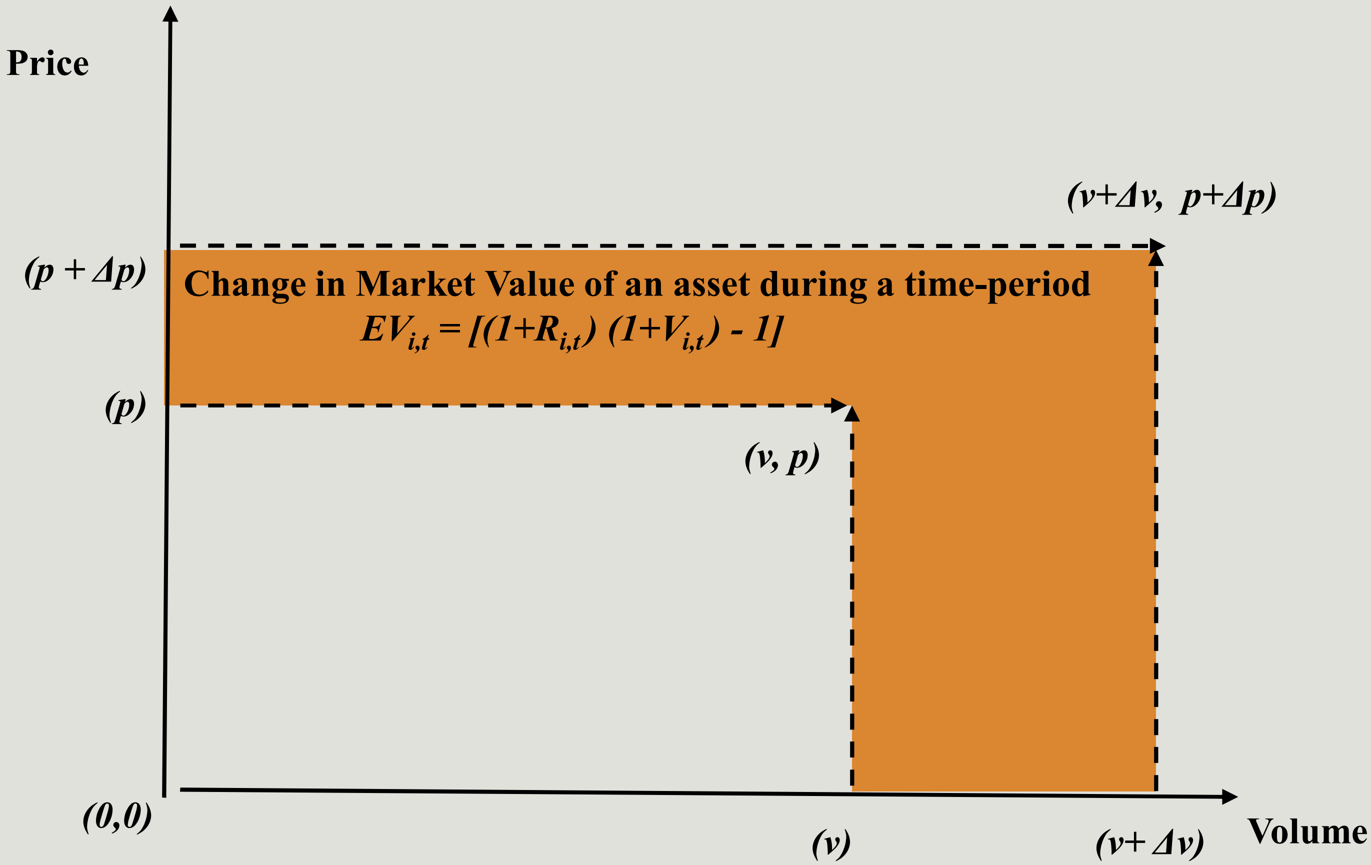
Names	Correlation (t-stats)	SSE
S1	99.38% (15.54)	7.10E-04
S2	99.32% (14.78)	7.77E-04
S3	99.51% (17.37)	3.08E-04
S4	99.48% (16.89)	2.85E-04
S5	98.94% (11.78)	2.33E-04
S6	98.72% (10.71)	4.65E-04
S7	95.48% (5.57)	4.43E-03
S8	98.73% (10.77)	1.10E-02
S9	92.42% (4.19)	1.57E-03
S10	99.74% (23.91)	5.14E-04
S11	97.05% (6.97)	3.30E-03
S19	99.82% (28.88)	6.98E-05
S20	92.90% (4.35)	2.54E-03
S21	99.73% (23.32)	9.94E-02

Charts of Average $V_{i,t}$ – Actual and Estimated




Portfolio Volume Changes for S9
Volume Changes
0.1
0.05
0
-0.05
-0.1
0.6
0.8
1
1.2
1.4
1.6
1.8
Risk
Actuals
RFM

Portfolio Volume Changes for S10
Volume Change
0.2
0.1
0
-0.1
-0.2
0.85
0.95
1.05
1.15
1.25
1.35
Risk
Actuals
RFM

Portfolio Volume Changes for S11
Volume Change
0.2
0.1
0
-0.1
-0.2
0.75
0.85
0.95
1.05
1.15
1.25
Risk
Actuals
RFM

Portfolios Volume Changes for S19
Volume Changes
0.07
0.03
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4
1.5
Risk
Actuals
RFM

Portfolio Volume Changes for S20
Volume Changes
0.2
0.1
0
-0.1
0.7
0.9
1.1
1.3
1.5
Risk
Actuals
RFM

Portfolio Volume Changes for S21
Volume Change
0.5
0
-0.5
-1
0.99
Risk
Actuals
RFM

Economic Implications



The investors who want to maximize wealth instead of just profits, should choose assets that give maximum Market Value for minimum variances in $R_{i,t}$ and $V_{i,t}$ for a given time period.

Results: Correlations and SSE between Actual Average $EV_{i,t}$ and Estimated Average $EV_{i,t}$ across P1 to P-full for the samples for which volume data were collected

Names	Correlation (%)	Correlation (t-stats)	SSE
S1	99.40%	15.73	7.28E-04
S2	99.33%	14.89	7.92E-04
S3	99.54%	17.92	3.18E-04
S4	99.51%	17.38	2.94E-04
S5	98.95%	11.85	2.33E-04
S6	98.73%	10.76	4.68E-04
S7	95.48%	5.56	4.44E-03
S8	98.73%	10.76	1.11E-02
S9	92.11%	4.10	1.54E-03
S10	99.74%	23.86	5.15E-04
S11	97.06%	6.98	3.30E-03
S19	99.82%	28.62	7.11E-05
S20	92.93%	4.36	2.55E-03
S21	99.75%	24.66	1.02E-01

Conclusions

1. Stock Returns **do not add linearly** in a Portfolio as they are Rational Functions. Hence we must model Prices using Linear Regression techniques and then calculate Average Stock Returns from the Price series.
2. The charts of the average returns indicate that the risk-return-efficient investments should be **carefully selected from such charts** as average returns plot nonlinear across risk and sometimes the lower risk assets offer higher returns.
3. Stocks can be **sorted** on various relevant financial parameters like size, profitability etc. and then the average returns for the portfolios should be estimated by the RF model.
4. However, for time series of Continuous Returns, the returns may be treated as **'approximately' linear** and modeled directly through multi-factor linear regressions.
5. **Change in stock volumes** are also important as they indicate the degree of realizability of the returns and even these can be estimated accurately using the RFM theory.
6. The wealth maximizing investors should choose assets that give maximum **Economic Value** for minimum β_1 and γ_1 for a given time period.
7. Price and Volume are **two complementary forces** of the market and the economic value of an asset flows through both of these factors.

Thus, the RFM theory, if used judiciously, can help the investors to make better and more economically efficient investments in the stocks and similar assets as compared to the existing asset pricing models.

Future Scope of Work

1. In-depth empirical analysis testing the RFM.

2. Identifying other relevant factors influencing the Stock Prices and Volumes along with their underlying theoretical rationales.

3. Further refinement of the RFM through careful mathematical modeling of the approximations that have been used here.

4. Comparison of the RFM with other asset pricing models.

Hopefully, there would be studies in the near future that would attempt to address these objectives.

Thank You